SECTION - H

DECISION THEORY

Decision Theory 10

This Module includes -

Decision Theory

SLOB Mapped against the Module

To appreciate quantitative tools for decision making in dynamic environment shrouded with risks and uncertainties. (CMLO 2a, 2b).

Module Learning Objectives:

After studying this module, the students will be able to

- Appreciate the decision making process in the context of ever changing business environment.
- Fathom the process of decision making under conditions of certainty, uncertainty and risk.
- Understand the basic aspects of Decision Tree.

Decision Theory

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Decision making is the most significant aspect of the management process. Efficacy of every aspect of management (planning, organizing, control, etc.) is pivoted on the effectivity of the decision making process. Effective decision making is linked to fulfilment of the objectives of the organization. An elaborately designed decision making process helps to make a more deliberate and effective decision. The steps of the process are discussed below:

Step 1: Identify the decision – it is important to identify the nature of decision that the decision maker is faced with. This paves way for making effective decisions.

Step 2: Gather relevant information - Before decision making, it is important to gather all relevant information. The source of information can be two types,

- Internal source- information available within the organisation.
- External source – information that are available beyond the scope of the organisation.

Step 3: Identify the alternatives – on the basis of the information collected the alternatives are zeroed upon. At this juncture it is important to make a list of all possible alternatives in order to make a correct and effective decision.

Step 4: Consider the evidence - In this step, the decision maker uses his knowledge and emotion to imagine what it would be like if one particular alternative is chosen and carried out. This would have to be thought about for all the possible alternatives. As the decision maker goes through this process (often with subtlety), he starts developing a notion as to which alternative results in the achievement of the organisational goal.

Step 5: Take action - In this step the decision maker is ready to make his call which is decided upon in the previous step.

Step 6: Review of the decision - After the above steps are undertaken and a decision is arrived at, the process of evaluation has to begin where the impact of the decision is considered. If the desired result is not achieved, the whole process has to be revisited.

The theoretical underpinnings of the decision making process is the subject matter of Decision Theory. The following aspects are noteworthy:

- Decision theory involves economic and statistical approaches for studying an individual's choices. Because it is based on ideas, attitudes, and wishes, analysts refer to it as a theory of choice.
- Decision theory enables the entity to make the most rational decision feasible in unknown and uncertain conditions, repercussions, and behaviours.
- In order to make better business decisions, companies worldwide use this theory to understand how customers and markets operate.

- Mathematicians, economists, marketers, data and social scientists, biologists, psychologists, philosophers, and politicians use two theory forms: normative and descriptive[1].

Though decision theory deals with the methods for determining the optimal course of action when a number of alternatives are available, given that the consequences cannot be forecast with certainty[2], for the purpose of this section of the study note, discussion is restricted to problems occurring in business, with consequences that can be described in Rupees of profit or revenue, cost or loss. For these problems, it is reasonable to consider that the best alternative is the one which results in the highest profit or revenue, or lowest cost or loss, on the average, in the long run.

Certainty[3], Uncertainty and Risk

It is obvious from the above discussion that the decision maker has to choose between alternatives. There are several possible alternatives (or outcomes) of an act (choosing between alternatives) which has to be endorsed in favour of, such that the end result, in terms of Rupees, is optimised. This is the fundamental aspect of decision theory. Now, the moot question is whether there is information regarding happening/ not happening of the outcomes. There can be as such two extreme cases; one where there is perfect information of happening/ not happening of the outcome. This is a situation where there is relevant past experience to enable statistical evidence for predicting the possible outcomes. This is the case of decision making under certainty. The other extreme is a situation where there are several possible outcomes, but there is no previous statistical evidence to enable the possible outcomes to be predicted. This is the case of decision making under uncertainty. In this particular situation probabilities cannot be assigned to the various states of nature (condition of happening/not happening of a particular state of the future). Thus there arises a third situation where there is predictability of happening/ not happening of future state of nature (condition of happening/not happening of a particular state of the future). This is the case of decision making under condition of risk. The terms 'risk' and 'uncertainty' are used interchangeably but there is significant difference between the two. A comparison chart is presented in the following lines which helps to understand the basic difference between the two. In making decisions under risk, the decision maker can predict the possibility of a future outcome, but when making decisions under uncertainty, the decision maker cannot. Risks can be managed while uncertainty is uncontrollable. The decision maker can assign a probability to risks events.

BASIS FOR COMPARISON	RISK	UNCERTAINTY
Meaning	The probability of winning or losing something worthy is known as risk.	Uncertainty implies a situation where the future events are not known.
Ascertainment	Measurable	Not Measurable
Outcome	Chances of outcomes are known.	The outcome is unknown.
Control	Controllable	Uncontrollable
Minimization	Yes	No
Probabilities	Assigned	Not assigned

Comparison Chart

[1] This aspect of the study of Decision Theory is beyond the scope of the study note. However, students may refer the website mentioned in parenthesis for in-depth understanding (https://www.wallstreetmojo.com/decision-theory/)

[2] It is important to note that most of the academic discussions on short term decision making (most of which is discoursed in Module 4 of this study note) occurs under conditions of certainty.

[3] It is important to note that the the distinction between risk and uncertainty is not essential for analysis in cost and management accounting and the terms are often used interchangeably in courses on Cost and Management accounting. But the distinction is essential part of Decision Theory as discussed in books on Operations Research. (Please refer Operations Research - An Introduction, Tenth Edition, by Hamdy A. Taha (Chapter 15: Decision Analysis and Games)).

In the following lines the conditions are introduced briefly:

(i) Decision making under conditions of certainty

Decision making is about selecting the best alternative from among an array of alternatives. The 'best' alternative refers to that particular alternative which helps a firm to maximise its profit[4] or minimise its cost. In decision theory, the alternatives are referred as acts and the possible events are referred as states of nature (outcomes of a random process).

The condition of certainty imply that the future is known and thus the probability of happening/ not happening of an event is one. All the models of short term decision making discoursed in Module 4 of this study note are under conditions of certainty[5]. In simple words, the decision-maker is conformed to what will happen when a decision is being made. It is a condition where the future is cent percent definite. This situation is conformed because of the availability of all reliable information. Thus the cause and effect are known with certainty. Due to known conditions, there are no conflicts in decision-making. This condition exists in routine decisions such as day-to-day activities, payment of wages, salaries, etc. Another example is when a person is going to buy a car, he can collect all the relevant information about that car, and he gets confirmed as to what type of car he is buying.

This is erroneous as the terms 'uncertainty' and 'future' are fabricated into each other. It is a well-accepted fact that the future is uncertain. In the ever changing business environment, even in the short term, nothing can be assumed to be cent percent assured, even under conditions of perfect information. As such, the notion of perfect information is also a misnomer.

The entire gamut of short term decision making that operate under conditions of certainty is often referred as the deterministic model. Though there are severe criticism for the models, they exist in the world of academia because of their simplicity and significant contribution to the knowledge base.

(ii) Decision making under condition of uncertainty

Uncertainty lies on the other end of the continuum[6]. In certainty, as discussed in the previous paragraph, the future is known and the decision maker, thus, need not worry about the happening /not happening of a particular state of nature as the future is cent percent assured. Whereas under condition of uncertainty, the future states of nature are unknown. There is no information available on the happening /not happening of the future state of nature. In decision making under uncertainty, the probability distribution associated with the states is either unknown or cannot be determined. This lack of information has led to the development of special decision criteria which would be discussed in brief, in later section of this module.

(iii) Decision making under condition of risk.

The term 'risk' is one of the most discoursed terms in finance literature. Various aspects of financial risk and its management is taken up for discussion in a later paper[7]. In simple terms, risk is a situation, where the decision maker is neither certain nor uncertain about the future states of nature. Thus there is imperfect information about the happening/ not happening of the future events. In mathematical terms, probabilities may be assigned to happening/ not happening of the future events. The probabilities may be either priori probabilities, derived from inherent symmetries, like the throw of a dice (the probability of throwing a four is one –sixth as there is one event favourable and six events that may occur) or statistical probabilities, obtained through analysis of homogenous data (for example, the chance of rain on 16th August 2022 may be assigned by looking into the weather report of last two hundred years. If rain had occurred on 60 days out of 200 days, then it may be said that the probability of rain on 16th August 2022 is 60/200=0.3 and the probability of rain not happing on 16th August 2022 is 0.7).

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4 In corporate finance, maximisation of shareholders' wealth (SWM) is preferred objective of the firm.

5 This is seldom true under conditions of long term where the impact of the uncertain future is obvious.

6 Today, the complex business environment is referred as VUCA environment which is an acronym for (V)Volatility, (U) Uncertainty, (C) Complexity and (A) Ambiguity. It applies to applied to assess the complex environments where tasks may vary and change as fast as the environment around them. Under such conditions the deterministic models (conditions of certainty) losses much of their lustre and decision making under uncertainty is considered as more significant.

7 Students may refer to Module 4 of paper 20A for in-depth knowledge of financial risk and various aspects of financial risk management.

Decision Making under Certainty

10.1

In decision theory, utility matrices are combined with various types of information about states of nature. The decision maker may or may not be able to gather prior information about the states of nature. In cases when only one state of nature is to be considered because of the nature of the problem or because of lack of information[8] are called decision-making under certainty. If the decision maker is certain as regards to the probability of happening/ not happening of an outcome, he is said to operate under condition of certainty. On the contrary, if the decision maker has imperfect information or no information about the happening/ or not happening of an event he is said to operate under conditions of uncertainty or risk. Thus it may be stated that in the realm of decision making, under condition of certainty each action will lead invariably to a specific outcome. In this situation only one state of nature exits and its probability is one. The following illustrates the understanding.

Solved Cases 1

Mr Pratap is considering setting up his stall in the playground in the evening of a particular day, say 20th August 2022. He has the option of selling ice creams or coffee. He has the option of buying Ice creams[9] from a whole seller @ ₹56 each and selling them for @ ₹60 each. Thus he would make a profit of ₹4 on each Ice cream cone. On a sunny day he sales 200 cones, but if it is a rainy day then sales fall and thus he is able to sell only 80 cones. On the contrary, he can sell coffee whereby he can make a profit of ₹6 per cup. On a sunny day he sales 100 cones, but if it is a rainy day then sales increases and thus he is able to sell 160 cups.

This can be represented in the following pay off matrix:

Particulars	States of Nature	
Acts	Sunny Day	Rainy Day
Sale Ice Cream	₹ 800[10]	₹ 320
Sale Coffee	₹ 600	₹ 960

Now, on the morning of 20th August 2022 he wakes up and find that it has been raining from the previous day night and for clarification he calls the met office and they confirm that it would rain for the whole day. Thus he faces a situation of certainty as there is only one state of nature and that is it being a rainy day. Thus his decision whether he would sell Ice cream or Coffee is based on one certain information and thus the above payoff matrix may be reduced to one with a single state of nature (Rainy Day).

[8] This is the case for various deterministic models of decision making like make or buy, shutting down of a product line, replacement of a machinery etc. This are discoursed in details in Module 4 of this study note.

[9] For simplicity it is assumed that only one type of ice creams is bought and sold namely, Strawberry ice creams in cones.

[10] This is calculated as profit per cone of ice cream (₹4) × number of cones sold (200). The other values in the payoff matrix are calculated in the same manner.

Particulars	States of Nature
Acts	Rainy Day
Sale Ice Cream	₹ 320
Sale Coffee	₹ 960

Thus Mr Pratap is better off if he sales Coffee on 20th August 2022, at the playground as the payoff (₹960) from selling coffee is higher than the payoff from selling ice cream. This is particularly because of the fact that it is known with certainty that the 20th August 2022 would be a rainy day.

Decision Matrix

The standard format for the evaluation of alternatives in decision theory is that of a decision matrix. In a decision matrix, the alternatives open to the decision-maker are tabulated against the possible states of nature. The alternatives (acts) are represented by the rows of the matrix, and the states of nature by the columns. The decision matrix is also referred as the payoff matrix when the cell values are presented in terms of net benefit. For the purpose of understanding a payoff table, or decision matrix, is shown below. The decision will be made among m of alternatives, identified as A_1, A_2, A_3…… A_m There may be more than one future "state of nature" N. The model considered here allows for n different futures. These future states of nature may not be equally likely, but each state will have some (known or unknown) probability of occurrence Since the future must take on one of the n values of the sum of the n values of must be 1.0. The outcome (or payoff, or benefit gained) will depend on both the alternative chosen and the future state of nature that occurs. For example, if the alternative A_i is chosen and state of nature N_j takes place (as it will with probability p_j), the payoff will be outcome O_{ij} A full payoff table will contain m times n possible outcomes.

Table: Decision Matrix

Alternative	States of Nature/Probability					
	N_1	N_2	-----	N_i	-----	N_n
	P_1	P_2	-----	P_i	-----	P_n
A_1	O_{11}	O_{12}	-----	O_{1j}	-----	O_{1n}
A_2	O_{21}	O_{22}	-----	O_{2j}	-----	O_{2n}
-----	-----	-----	-----	-----	-----	-----
A_i	O_{il}	O_{i2}	-----	O_{ij}	-----	O_{in}
-----	-----	-----	-----	-----	-----	-----
A_m	O_{ml}	O_{m2}	-----	O_{mj}	-----	O_{mn}

Under condition of certainty there would be only one state of nature as the future is known with certainty. Thus the decision matrix or payoff matrix would be as given below.

Pay off Matrix (under condition of certainty)

Alternative	State of nature ($p_i = 1$)
A_1	O_1
A_2	O_1
A_3	O_1
A_4	O_1
A_m	O_m

Decisions Making under Risk

10.2

In finance literature, there are various connotation of the term 'Risk'. The most frequently used has a negative, 'a condition in which there is a possibility of an adverse deviation from a desired outcome'.

In a risky environment, the decision maker operates under condition of imperfect information. A manager may understand the problem and the alternatives, but has no guarantee how each solution will work. This is a fairly common condition under which the decision maker operates.

When new and unfamiliar problems arise, non- programmed decisions are specifically tailored to the situations at hand. The information requirements for defining and resolving non-routine problems are typically high. Although computer support may assist in information processing, the decision will most likely involve human judgment. Most problems faced by higher-level manager's demand non-programmed decisions. This fact explains why the demands on a manager's conceptual skills increase as he or she moves into higher levels of managerial responsibility.

When a manager lacks perfect information risk arises. Under a state of risk, the decision maker has incomplete information about available alternatives but has a good idea of the probability of outcomes for each alternative. While making decisions under a state of risk, managers must determine the probability associated with each alternative on the basis of the available information and his experience.

In decision theory, under risk the decision maker assumes that there exist a number of possible future states of nature as is presented in the previous table. Each has a known (or assumed) probability of occurring, and there may not be one future state that results in the best outcome for all alternatives Examples of future states and their probabilities are as follows:

- Weather will affect the profitability of alternative construction schedules. In this case the probabilities of rain and of good weather can be estimated from historical data.
- Alternative economic futures (boom or bust) determine the relative profitability of conservative versus high-risk investment strategy. In this case the assumed probabilities of different economic futures might be based on the judgment of the experts.

Probabilities

Probabilities are mathematical expression which is used to denote the likelihood that an event or state of nature will occur. It is expressed in decimal and varies between 0 and 1. When the probability of occurrence of an event is 0, it denotes nil likelihood of occurrence whereas a value of 1 signifies absolute certainty – a definite occurrence. For example, the chance that 8 would come up in a throw of a dice is 0 as there are no faces of 8 in a dice. And the chance that a head or a tail would come up in a throw of a coin is 1 as those are the two events possible in a throw of a coin[11]. A probability of 0.6 means that the event is expected to occur six times out of ten.

[11] Ignoring the chance that the coin would land vertically on its edge in a throw.

The total of the probabilities for events that can possibly occur must sum to 1.0. For example, if a tutor indicates that the probability of a student passing an examination is 0.6 then this means that the student has a 60 per cent chance of passing the examination. Given that the pass/fail alternatives represent an exhaustive listing of all possible outcomes of the event, the probability of not passing the examination is 0.4. The information can be presented in a probability distribution. A probability distribution is a list of all possible outcomes for an event and the probability that each will occur. The probability distribution for the above illustration is as follows:

Outcome	Probability
Pass examination	0.6
Do not pass examination	0.4
Total	1.0

Objective and Subjective Probabilities

Probabilities are categorised either as objective and subjective. Glyn A. Holton[12] posits 'according to objective interpretations, probabilities are real. We may discover them by logic or estimate them through statistical analyses. According to subjective interpretations, probabilities are human beliefs'. Objective probabilities are either pre-defined or are arrived at from statistical inferences. This is corroborated in the works of Frank Knight who in 1921 stated that probabilities (objective) may be obtained in two manners:

- A priori probabilities are derived from inherent symmetries, as in the throw of a die.
- Statistical probabilities are obtained through analysis of homogenous data.

As such, objective probabilities are established mathematically or compiled from historical data. Tossing a coin and throwing a dice are examples of objective probabilities. For example, the probability of heads occurring when tossing a coin logically must be 0.5. This can be proved by tossing the coin many times and observing the results. And the chance of getting a 1 in a throw of dice is 1/6.

In business decisions, the probabilities (chances of a particular state of nature of happening/not happening) are often estimated based on managerial judgement. Probabilities established in this way are known as subjective probabilities because no two individuals will necessarily assign the same probabilities to a particular outcome. Subjective probabilities are also known as uncertainty16 and are based on an individual's perspectives of future events and their impact on operations of the entity.

Two important aspects of Probability

When the weather forecaster says there is a 40% probability that it will rain today, it also implicitly means there is a 60% probability that it will not rain. This illustrates the two basic requirements of probability:

1. The probability values assigned to each of the possible outcomes must be between 0 and 1; and
2. The probable values assigned to all of the possible outcomes must total 1.

Independent Events and Mutually Exclusive Events Independent Events

If the occurrence or non-occurrence of one event does not change the probability of the occurrence of the other event, the two events are said to be independent.

The addition law can be used when there are two possible events and we want to know the probability that at

[12] A comprehensive understanding about risk and uncertainty is presented in a conceptual paper titled 'Defining Risk – Perspectives' (available at https://www.glynholton.com/wp-content/uploads/papers/risk.pdf).

least one of the events will occur. In other words, for events A and B, we want to know the probability that event A or event B or both events will occur.

Events that are independent and not mutually exclusive can have sample points in common. That is, in some cases both A and B can occur. We need to include those cases in our calculation of the probability that at least one of the events will occur; but we do not want to double count them because of counting them once with A's probability and again with B's probability.

The union of events A and B is the event containing all the sample points belonging to A or B or both. It represents the probability that either A or B will occur, including the probability that both will occur.

Calculating the Joint Probability of two Independent Events

The area of the overlap—the joint probability—is the probability that both events will occur. That area qualifies to be included in the probability that either one of the events will occur, because one of the events certainly occurs in the area of the overlap. But we want to include it once, not twice, so we subtract it from the sum of the two events' probabilities.

For example, if the probability of Event X occurring is 20% and the probability of Event Y occurring is 25% and they are independent and not mutually exclusive events, the probability of both X and Y occurring is 0.20 × 0.25, or 0.05 or 5%.

Mutually Exclusive Events

If events are mutually exclusive, it means that if one of them occurs, the other event cannot occur. Either one or the other can occur but not both.

Solved Cases 2 (Independent Events and Mutually Exclusive Events)

In its sales forecasting, an appliance retailer develops a set of probabilities for sales in each of its product lines for the coming year. Sales forecasts for two of these product lines are as follows:

Refrigerators: There is a 30% probability that sales of refrigerators will be ₹50,00,000; a 50% probability that sales will be ₹75,00,000; and a 20% probability that sales will be ₹1,00,00,000.

Electric Ranges: There is a 25% probability that sales of electric ranges will be ₹20,00,000; a 55% probability that sales will be ₹30,00,000; and a 20% probability that sales will be ₹ 50,00,000.

The forecasts for these appliances relate to sales for the following year. Therefore, the actual events (sales of refrigerators and ranges) will both be occurring at the same time. The forecast for sales of refrigerators is not dependent on sales of electric ranges occurring, and the forecast for sales of electric ranges is not dependent on sales of refrigerators occurring. Thus sales of refrigerators and sales of ranges are independent of each other.

What is the probability that sales of refrigerators will be ₹75,00,000 or sales of electric ranges will be ₹30,00,000 next years? According to the above information:

(i) The probability that sales of refrigerators will be ₹75,00,000 next year is 50%.

(ii) The probability that sales of electric ranges will be ₹30,00,000 next year is 55%.

(iii) The probability that sales of refrigerators will be ₹75,00,000 and that sales of electric ranges will be ₹30,00,000 is 0.50 × 0.55, which equals 0.275 or 27.5%.

Therefore, the probability that sales of refrigerators will be ₹75,00,000 or sales of electric ranges will be ₹ 30,00,000 next year or that both events will occur next year is 0.50 + 0.55 − 0.275 = 0.775 or 77.5%

In the example above, refrigerator sales of ₹75,00,000 and electric range sales of ₹ 30,00,000 are not mutually

exclusive. In other words, it is possible for refrigerator sales to be ₹75,00,000 and for electric range sales to be ₹ 30,00,000. In fact, we calculated the probability of that occurring as 27.5%.

What if instead the retailer wanted to know the probability of refrigerator sales being either ₹50,00,000 or ₹75,00,000? That makes our probability question one of mutually exclusive events. Refrigerator sales cannot be ₹50,00,000 and ₹75,00,000 at the same time.

(i) The probability that sales of refrigerators will be ₹ 50,00,000 next year is 30%.

(ii) The probability that sales of refrigerators will be ₹ 75,00,000 next year is 50%.

Therefore, the probability that sales of refrigerators will be ₹ 50,00,000 or ₹ 75,00,000 next year is 0.30 + 0.50 = 0.80 or 80%

Note: Independent events and mutually exclusive events are very different.

Two events A and B are independent if the occurrence or non-occurrence of one event does not change the probability of the occurrence of the other event.

Two events A and B are mutually exclusive if only one of them can occur, that is, when one of them occurs, the other event cannot occur.

Dependent Events and Conditional Probability

When there are two events, A and B, and the occurrence of B depends upon the occurrence of A, the probability that both events will occur is the probability that the first event will occur, multiplied by the conditional probability that the second event will occur given that the first event has already occurred.

Three Methods of Assigning Probable Values

Three methods are used to assign probable values to possible outcomes: The Classical Method, the Relative Frequency Method, and the Subjective Method.

1. **Classical Method:** This method assumes that each possible outcome has an equal probability of occurring. Thus, if there are ten possible outcomes, each outcome is assumed to have a 10% probability of occurring. This is the method used to assign probabilities to coin tosses or dice rolls. Business decisions don't usually involve coin tosses or dice rolls, so the classical method is seldom used in situations of business uncertainty.
2. **Relative Frequency or Objective Method:** When factual information is available that can be used to determine the probability of something occurring; the use of that information to assign probabilities is called the relative frequency method. The information may come from a sample, analytical data, or any other reliable source.
3. **Subjective Method:** This method is used when neither the classical nor the relative frequency methods can be used because the possible outcomes are not equally likely and relative frequency da-ta is not available. With the subjective method of assigning probabilities, we use whatever data is available and add to that data our own experience and intuition. After considering all available information, we assign a probable value that expresses our degree of belief that the outcome will occur. Subjective probability is personally determined, and different people will assign different probabilities to the same event. Despite this relative freedom in assigning probabilities, the two necessary requirements for all probabilities must nevertheless be met:

   (i) The probable value for each possible outcome must be between 0 and 1; and

   (ii) All the probabilities for all the possible outcomes must total 1.

   Sometimes the various methods are used in combination, such as when probabilities are determined by combining estimates from the classical or relative frequency methods with subjective probability estimates.

Expected Value (or Expected Return)

The concept of expected value is very important. The expected value of an action is found by multiplying the probability of each potential outcome by its payoff. Therefore, expected value, or expected return, is a weighted average of the possible returns, with the weights being the probabilities of occurrence. The expected value of a discrete random variable is the weighted average of all the possible values of the random variable. The weights are the probabilities for each of the values. The expected value is the mean value, also known as the average value.

A weighted average can be calculated only for discrete probability distributions. It is not possible to calculate a weighted average for a continuous probability distribution because the number of possible variables is infinite.

The general model of decision making under risk when probabilities may be assigned in an objective manner to the states of nature is through the Expected Value criterion. In the below mentioned lines a Caselet is furnished for conceptualisation of the model.

Solved Cases 3

Subbuji is a small vendor who is undecided on what to sell in the fairground as a fair is to be organized in ten days. He has the option of selling tea or ice creams on a day one (16th August 2022) of the fair. He has made a projection that selling tea would fetch him a profit of ₹300 if it rains on 16th August 2022, but if it is sunny and humid on the day, he would not have much customers and then he would make a profit of ₹30. If he sells ice cream his profit is much higher (₹150) if 16th August 2022 is hot and humid, but if he decides to sell ice cream and it rains on that day then his profit would be ₹10. How would he make the decision of what to sell (tea or ice cream) on 16th August 2022?

Solution:

Subbuji has to decide on selling tea or ice – cream on 16th August 2022. These are termed as acts. On 16th August 2022 when these acts are to take place it can either be sunny and humid or rainy. These are called states of nature. Subbuji's decision (to sell tea or ice – cream) is reliant on the information he can garner on the states of nature. The information on chance of the day being 'sunny and humid' or being 'rainy' can be got from data of the last years. Subbuji can visit the met office and check the data of the last 200 years[13]. This would mean 200 data points about whether 1st June 2022 would be 'sunny and humid' or 'rainy'. After collection of the data, Subbuji finds that out of 200 days (past data) it rained for 30 days. From this he can infer that the probability of rain on 16th August 2022 as 0.15 (30/200). And the probability of the day being 'sunny and humid' is 0.85 (1 – 0.15). Once Subbuji has got this information about the state of nature he can frame the expected pay off matrix and take his decision based on expected value criterion.

Probability of Occurrence		States of Nature	
		Hot and Humid	Rainy
		0.85	0.15
Acts	Sell Tea	30	300
	Sell Ice-Cream	150	10
The 2×2 Matrix (for expected value[14] calculation)			

[13] Any number of years is possible. And it is to be noted that greater the data, higher is the accuracy of statistical inference. But collection of more data involves more cost. Thus it is to be noted that there is a cost of information. And there is a trade-off between more data and more cost of gathering data.

[14] Expected Value of an Opportunity (EV) is a term used to describe the expected value of a business opportunity.

The expected value (on the basis of which the decision is to be taken) is given as

$EV = \sum P(X_i) \times X_i$

Where $P(X_i)$ = Probability of occurrence of event i and X_i is the payoff related to the event i

in the given case, the EV (tea) = 30 × 0.85 + 300 × 0.15 = 70.5 and

EV (ice –cream) = 150 × 0.85 + 10 × 0.15 = 129.

Since this is pay off, Subbuji would choose that act which gives the highest pay off. Thus Subbuji takes his decision about which act to consider based on the highest expected value in case of pay off. This is only possible if the decision maker has access to information about the probability of occurrence of the various states of nature. Such a situation is referred as a risky situation. Information about the probability of occurrence of the state of nature is got either through statistical inference as Subbuji did, or are priori (defined from previous like in a throw of dice).

Other aspects of decision making under conditions of risk are discoursed in details in later section of this study note.

However, in a very real sense risk does not always implicitly carry a negative connotation. Where investments are concerned (both capital investments and security investments), risk is the possibility that an investment's actual return will differ from its expected return. This difference may be either positive or negative.

Investment Appraisal and Risk[15]

Condition of risk denotes situation where the decision maker have information about happening/ not happening of an event. The decision maker denotes the likelihood of happening/ not happening of an event in terms of probabilities. For example, a decision maker can assign 70 per cent probability that returns from a project will be in excess of ₹1,00,000. This also means that there is a 30 per cent probability that returns will be less than ₹1,00,000.

Risk for an investment can be measured by the variability, or dispersion, of its potential returns around the mean return. The mean return is given by the expected value of the returns. The variance and the standard deviation of a set of potential returns are measurements of their dispersion about the mean. Thus, the risk of an investment is measured by the variance and standard deviation of its potential returns.

In everyday usage the terms risk and uncertainty are not clearly distinguished. If one is asked for a definition, one should not make the mistake of believing that the latter is a more extreme version of the former. It is not a question of degree; it is a question of whether or not sufficient information is available to allow the lack of certainty to be quantified.

Such type of environment is very sure and certain by its nature. This means that all the information is available and at hand. Such data is also easy to attain and not very expensive to gather.

So the manager has all the information he may need to make an informed and well thought out decision. All the alternatives and their outcomes can also be analysed and then the manager chooses the best alternative.

Another way to ensure an environment of certainty is for the manager to create a closed system. This means he will choose to only focus on some of the alternatives. He will get all the available information with respect to such alternatives he is analysing. He will ignore the other factors for which the information is not available. Such factors become irrelevant to him altogether.

[15] This aspect is discoursed in details in Module 2 of Paper 14. For comprehensive understanding of the topic, students may look into the said module.

Standard Deviation and Variance as a Measure of Risk

The variance and standard deviation both give an idea of the variability of the possible values about the mean. The variance and the standard deviation measure how far from the mean (the expected value) the various possible values lie. The variance is used to summarize the variability in the values of a random variable. Another word for this variability is dispersion. The amount of variability in the values of the random variable around their mean (or average) is the amount by which they are dispersed, or the amount of their dispersion. The amount of dispersion is important because it is a measurement of risk. The greater the dispersion of the values around their mean, the greater the risk associated with the values because there is a larger chance that the actual results will be different from the expected value. If the values are highly dispersed about their mean, then they vary widely from their expected value.

The variance of a population is represented by σ^2 (sigma squared). The variance is the sum of the squares of all the differences or deviations from the mean (average), weighted according to their probabilities. The difference from the mean of each result is important because it indicates the distance that particular measurement is from its expected value. The variance is actually a weighted average of the squared deviations. The standard deviation is the positive square root of the variance. It is represented by σ (sigma).

Variation within the possible cash flows for each project is also important because a project with a high variability of cash flows has more risk than a project for which all the possible cash flows are close together.

The standard deviation of the probability distribution of these subjectively determined potential cash flows expresses the dispersion, or variability, of possible returns around the expected return. If the standard deviation is large, it means the variability of returns is large and the risk of the project is higher. Thus, standard deviation is a measure of risk. By expressing differences from the expected return in terms of numbers of standard deviations from the mean (expected return), we can state the probability that the actual return will fall within an interval relative to the mean, or expected return. The greater the standard deviation, the greater is the potential for loss or gain.

Standard deviation is always expressed in the same units as the distribution. Thus, if the distribution is a distribution of annual rates of return on an investment, the returns and the standard deviation of the returns are both expressed as annual percentages. If the distribution is a distribution of annual cash flows in units of currency, both the cash flows and the standard deviation of the cash flows will be expressed as currency amounts.

The variance and standard deviation of each of the expected cash flows is calculated in the same way as the variance and standard deviation. The narrower the distribution of the data, the lower the standard deviation will be. The lower the standard deviation, the lower the risk. The wider the distribution of data, the higher the standard deviation and the higher the risk. Standard deviation is a measure of the dispersion of a probability distribution and thus a measure of the riskiness of a project.

Again coefficient variation also measures the risk. Coefficient of variation is the relative measure of dispersion. It measures the standard deviation relative to the mean in percentages. The coefficient of variation is calculated simply by dividing the standard deviation by the expected return (or mean):

$$\text{Coefficient of variation} = \frac{\text{Standard Deviation of Return}}{\text{Expected Return}}$$

For example, assume that investment in financial instrument A has an expected return of 20% and a standard deviation of 15%, whereas investment in financial instrument B has an expected return of 25% and a standard deviation of 20%. The coefficients of variation for the two investments will be:

$$\text{Coefficient of variation}_{(A)} = \frac{\text{Standard Deviation of Return}}{\text{Expected Return}} = \frac{15\%}{20\%} = 0.75$$

$$\text{Coefficient of variation}_{(B)} = \frac{\text{Standard Deviation of Return}}{\text{Expected Return}} = \frac{20\%}{25\%} = 0.80$$

The interpretation of these results would be that investment in financial instrument A is less risky, as the coefficient of variation of the investment is lower. Another test statistic relating to dispersion is the standard error which is a measure often confused with standard deviation. Standard error is the measure of variability of a sample, used as an estimate of the variability of the population from which the sample is drawn. When we calculate the sample mean, we are usually interested not in the mean of this particular sample, but in the mean of the population from which the sample comes. The sample mean will vary from sample to sample and the way this variation occurs is described by the 'sampling distribution' of the mean. We can estimate how much a sample mean will vary from the standard deviation of the sampling distribution. This is called the standard error (SE) of the estimate of the mean.

Illustration 1

We are comparing two investment projects. Both have expected returns of 20%, but the standard deviation of Project A's returns is 15%, while the standard deviation of Project B's returns is 9%. Which one is relatively riskier?

Solution:

CV of Project A = 0.15 ÷ 0.20 = 0.75

CV of Project B = 0.09 ÷ 0.20 = 0.45

Because it has a higher Coefficient of Variation (CV), Project A is the relatively riskier project.

Illustration 2

Two investments have different expected returns. Project A's expected return is 20% and the standard deviation of its returns is 15%. Project B's expected return is only 10%, while the standard deviation of its returns remains at 9%. Which project is relatively riskier?

Solution:

CV of Project A = 0.15 ÷ 0.20 = 0.75

CV of Project B = 0.09 ÷ 0.10 = 0.90

Because Project B's expected return has decreased from 20% to 10%, as compared to example 1, above Project B's coefficient of variation has increased from 0.45 to 0.90. Therefore, Project B is now the relatively riskier project.

Illustration 3

You are required to select from the following two Projects, which are mutually exclusive:

Project X:

Estimated Net Cash Flows (₹)	Probability
2,000	0.3
3,000	0.4
4,000	0.3

Project Y:

1,000	0.2
2,000	0.2
3,000	0.2
4,000	0.2
5,000	0.2

The Expected value of both the Projects = ₹3,000

Solution:

Since, Expected Value of both the Projects are same, hence, we are required to compute Standard Deviation and Co-efficient of Variations of both the Projects:

Computation of SD of Project X

Cash Flow(X) ₹	Probability (p)	EV (Xp) ₹	X- $\bar{X}$ Variance	p (X- $\bar{X}$)^2 ₹
2,000	0.3	600	-1,000	3,00,000
3,000	0.4	1,200	0	0
4,000	0.3	1,200	+1,000	3,00,000
	EV	$\bar{X}$ = 3,000		6,00,000

SD of Project X = ₹ 775 $\left(\sqrt{p(X - \bar{X})^2}\right)$

On the basis of similar calculations, the SD of Project Y = ₹ 1,414. $\left(\sqrt{p(Y - \bar{Y})^2}\right)$

Decision:

Project X is selected, since SD is less, having less variability.

We can also calculate Co-efficient of Variation (CV):

Project X = (SD ÷ Mean) × 100 = (₹ 775 ÷ ₹ 3,000) × 100 = 25.83 %

Project Y = ₹ 1,414 ÷ 3,000 = 47.13 %.

Decision:

Project X is selected, since its CV is less.

Expected Value in Estimating Future Cash Flows

Expected value is a term that means a weighted average of the possible values using the probabilities as the weights. Any time the word "expected" is used in the context of an "expected value" or an "expected cash flow," it refers to the idea of calculating a weighted average of the possible values using the probability of each value as its weight.

Estimating, or projecting, future cash flows are an important application of expected value. It is used in capital budgeting analysis for evaluating potential projects. It is important to know how to calculate estimated future cash flows from a potential project for use in a capital budgeting analysis.

A budgeted amount of future cash flow is often thought of as an absolute number. Unfortunately, though, future cash flows cannot be accurately ascertained because there are many events that can affect a project's net cash flows. Every project has numerous possible future cash flows. A project has a range of estimated cash flows that reflect different possibilities that management can foresee.

In determining the various possible cash flows, management must:

1) To determine which influences have affected the net cash flows of similar projects in the past, such as economic conditions, labour conditions, or international conditions, and then

2) To make assumptions about each of those events and the manner in which those events might affect the project. For instance, if a recession is expected, management might assume that demand for the project's product will be below normal.

3) Once these specific assumptions have been formulated, the management accountant then estimates the impact that each assumption could have on the net cash flow in each year of the project's life. The manager develops several possible cash flow levels for each year, along with probabilities of each cash flow level occurring. This will be a discrete probability distribution (not a continuous one), and the probabilities for each year will all sum to 1 or 100%.

4) The management accountant will then calculate the expected value for the net cash flow for each year of the project's life by calculating the weighted average of all the possible cash flows for each year.

5) These calculated expected values of future cash flows will be the cash flows used in the capital budgeting analysis for each year.

Approaches have been developed to choose the best option when the decision maker has several alternatives and there is uncertainty with respect to future events.

Decision Models

Let us consider the case of a manufacturing company, which is interested in increasing its production to meet the increasing market demand. The following steps are required to be considered in the context of Decision Model:

Step I. To determine all possible alternatives

The first obvious step involved before making a rational decision is to list all the viable alternatives available in a particular situation. The following options are available to the manufacturer:

(a) To expand the existing manufacturing facilities (Expansion);

(b) To setup a new plant (New facilities);

(c) To engage other manufacturers to produce for him as much as is the extra demand (Sub- contracting).

Step II. To identify the future scenario

It is very difficult to identify the exact events that may occur in future. However, it is possible to list all that can happen. The future events are not under the control of the decision-maker. In decision theory, identifying the future events is called the state of nature. In the case which we have taken of a particular manufacturing company, we can identify the following future events:

(a) Demand continues to increase (High demand)

(b) Moderate demand

(c) Demand starts coming down (Low demand)

(d) The product does not remain in demand (No demand).

Step III. To prepare a pay-off table

The decision-maker has to now find out possible payoffs, in terms of profits, if any, of the possible events taking place in future. Putting all the alternatives together (Step I) in relation to the state of nature (Step II) gives the payoff table. Let us prepare the payoff table for our manufacturing company.

	State of nature			
	High Demand	Moderate Demand	Low Demand	No Demand
Expansion	1	2	3	4
New Facilities	5	6	7	8
Sub-Contact	9	10	11	12

Pay-off Table

If expansion is carried out and the demand continues to be high (one of the 12 alternatives), the payoff is going to be maximum in terms of profit of say ₹ X. However, if expansion is carried out and there is no demand (situation 4), the company will suffer a loss.

Step IV. Selecting the best alternative

The decision-maker will, of course, select the best course of action in terms of payoff. However, it must be understood that the decision may not be based on purely quantitative payoff in terms of profit alone, the decision-maker may consider other qualitative aspects like the goodwill generated which can be encashed in future, increasing market share with an eye on specially designed pricing policy which ultimately gives profits to the company, etc.

Decision Making with Probabilities

If a decision maker can estimate the probabilities of the future events, these should be incorporated into the decision model. In the steps in constructing payoff tables or decision trees, probabilities are used in determining payoffs. A common approach to decision making under uncertainty is the expected value criterion. The expected value (EV) of a decision alternative is calculated as follows:

EV (alternative) = (probability of first state of nature) × (outcome of that state of nature) + (probability of second state of nature) × (outcome of the second state of nature) + . . . for all states of nature.

In essence, the EV represents a weighted average of the outcomes, using probabilities as weights. The alternative selected is the one with the highest EV for maximization problems and the lowest EV for minimization problems.

Illustration 4

Building Ltd. owns land in Noida and intends to build a condominium development on the site. The company is deciding on whether to build a small, medium or large development. Demand is uncertain and fluctuates; demand could be low, medium or high. Management at Building Ltd. has determined profit payoffs will be:

	Demand	(all amounts in ₹ 000s)
Alternatives	Low	Medium	High
Small D_1	1,400	1,400	1,400
Medium D_2	1,100	1,600	1,600
Large D_3	(1,300)	1,200	2,100

Management has determined the probabilities of demand to be:

Low = P (low) = 0.20

Medium = P (medium) = 0.35

High = P (high) = 0.45

Solution:

The expected value of each alternative is calculated as:

Alternatives:

Small EV = 0.2(1400) + 0.35(1400) + 0.45(1400) = ₹ 1,400

Medium EV = 0.2(1100) + 0.35(1600) + 0.45(1600) = ₹1,500

Large EV = 0.2(-1300) + 0.35(1200) + 0.45(2100) = ₹ 1,105

Conclusion

Using the expected value criterion and absent of any qualitative considerations, the best alternative is to build a medium condominium complex as this provides the highest expected value.

Illustration 5

The following information is available for a Company:

Sales Volume (units)	Probability (%)
10,000	10
12,000	15
14,000	25
16,000	30
18,000	20

Projected sales and costs are as under:

Sales Price per unit: ₹ 6; Variable Cost per unit: ₹ 3.50; Fixed Costs: ₹ 34,000

Required:

(i) Probability that the Company will at least Break-even

(ii) Probability that the Profit will be at least ₹ 10,000.

Solution:

(i) Contribution per unit = ₹ 2.50 (₹ 6 - ₹ 3.50)

BEP (units) = Total Fixed Costs ÷ Contribution per unit = ₹ 34,000 ÷ ₹ 2.50 = 13,600 units.

The probability that at least Break-even = 0.25 + 0.30 + 0.20 = 0.75 = 75%.

(ii) The Profit will be at least ₹ 10,000:

Then, BEP (units) = ₹ 34,000 + ₹ 10,000 ÷ ₹ 2.50 = 17,600 units.

The required Probability = 20%

Limitations of the Expected Value Model:

- Not all future events are foreseeable and, therefore, may be omitted from the model
- The model assumes future events are independent of each other. There can be overlap between future events.
- It is difficult to accurately assess the probability of future events.
- The model ignores qualitative considerations in making a decision.
- The model ignores the decision maker's attitude towards risk. The expected value model assumes the decision maker is risk neutral. If the decision maker is risk seeking or risk averse, both the expected value of the decision and its dispersion become relevant in choosing the best decision.

Expected Value of Perfect Information (EVPI)

If we assume that an economic forecasting service is available that can reveal the future state of the economy with absolute certainty. This service has a proprietary computer model that has never been wrong, but the service costs ₹ 35,000. Should we purchase it?

Perfect information is knowledge about the future that would enable us to make the best choice today for any possible situation in the future. If we knew in advance the future state of the economy, we could make a much more informed choice between say, Project X and Project Y.

As we calculate the expected value of this perfect information, keep in mind that we do not know in advance what the perfect information will be. In other words, we must determine what it would be worth to us to know this perfect information before we know what we are buying.

Companies can sometimes obtain information that reduces or eliminates the uncertainty associated with the different future events/states of nature of a problem. The EVPI refers to the maximum amount a company would pay to obtain this information.

Formula to calculate EVPI:

EVPI = EV of best alternative with perfect information - EV of best alternative without perfect information

Illustration 6

In Illustration 4, the best alternative was to build a medium condominium complex as this resulted in the highest expected value (EV = ₹1,500). If perfect information were available, that is, the probabilities were known with certainty, the optimal decision strategy is:

If low demand occurs, build a small complex.

If medium demand occurs, build a medium complex.

If high demand occurs, build a large complex.

Findout EVPI.

Solution:

The expected value of this optimal decision strategy is:

0.20 (1,400) + 0.35(1,600) + 0.45(2,100) = ₹1,785

Therefore, the EVPI is:

EVPI = EV of best alternative with perfect information - EV of best alternative without perfect information

= ₹1,785 - ₹1,500 = ₹ 285

Limitations of EVPI

Perfect information is rarely, if ever, available. When determining whether to obtain additional information, the decision maker must weigh the additional expected value arising from perfect information against the costs of obtaining this information.

Pay-off tables

Pay-off tables identify and record all possible outcomes (or pay-offs) in situations where the action taken affects the outcomes.

Illustration 7

ABC Company Co is trying to set the sales price for one of its products. Three prices are under consideration, and expected sales volumes and costs are as follows.

Price per unit	₹4	₹4.30	₹4.40
Expected sales volume (units)			
Best possible	16,000	14,000	12,500
Most likely	14,000	12,500	12,000
Worst possible	10,000	8,000	6,000

Fixed costs are ₹ 20,000 and variable costs of sales are ₹ 2 per unit.

Which price should be chosen?

Solution:

Here we need to prepare a pay-off table showing pay-offs (contribution) dependent on different levels of demand and different selling prices.

A. Price per unit	₹ 4	₹4.30	₹4.40
B. Contribution per unit (A - ₹2)	₹2	₹2.30	₹2.40
C. Total contribution towards fixed costs (₹) (B × units):			
Best possible	32,000	32,200	30,000
Most likely	28,000	28,750	28,800
Worst possible	20,000	18,400	14,400

(a) The highest contribution based on most likely sales volume would be at a price of ₹4.40 but arguably a price of ₹4.30 would be much better than ₹4.40, since the most likely profit is almost as good, the worst possible profit is not as bad, and the best possible profit is better.

(b) However, only a price of ₹4 guarantees that the company would not make a loss, even if the worst possible outcome occurs. (Fixed costs of ₹20,000 would just be covered.) A risk averse management might therefore prefer a price of ₹4 to either of the other two prices.

Decision Making under Uncertainty

10.3

In the absence of homogenous data, neither priori probabilities nor statistical inferences can be used to define an opinion about a data set. Frank Knight (1921) used the term 'measureable uncertainty' to describe opinions based on probabilities. On the other, he used the term 'unmeasurable uncertainty' to describe opinions based on human judgements[16]. In simple terms, situations where objectives probabilities cannot be assigned to the states of the nature as no prior information is available gives rise to the condition of decision making under uncertainty.

Uncertainty, in common parlance, is a state of not knowing whether a proposition is true or false. Suppose Mr ASA went to a casino. There the dealer is about to roll a dice. If the result is a six, Mr ASA is going to lose ₹100.

What is Mr ASA's risk? What, is the subjective opinion (subjective probability) that Mr ASA will lose ₹100?

It may seem to be one chance in six (which is a general answer). But it is not known from previous how may sides the dice have. The information that the die is 10 sided one changes the perspective about probability of throwing a six. This example illustrates how one can be uncertain but not realize it. To clarify, an individual is uncertain of a proposition if she

- does not know it to be true or false or
- is oblivious to the proposition.

Probability is often used as a metric of uncertainty, but its usefulness is limited. At best, probability quantifies perceived uncertainty.

A decision problem, where a decision-maker is aware of various possible states of nature but has insufficient information to assign any probabilities of occurrence to them, is termed as decision-making under uncertainty. A decision under uncertainty is when there are many unknowns and no possibility of knowing what could occur in the future to alter the outcome of a decision.

The decision maker feels the uncertainty about a situation when he can't predict with complete confidence what the outcomes of the actions will be. The decision maker experiences uncertainty about a specific question when he can't give a single answer with complete confidence.

Launching a new product, a major change in marketing strategy or opening the first branch could be influenced by such factors as the reaction of competitors, new competitors, technological changes, changes in customer

[16] The famous definition of Frank Knight (1921) reads; 'to preserve the distinction . . . between the measurable uncertainty and an unmeasurable one we may use the term "risk" to designate the former and the term "uncertainty" for the latter'. This statement is Knight's famous definition of risk. Risk relates to objective probabilities. Uncertainty relates to subjective probabilities. (available at https://www.glynholton.com/wp-content/uploads/papers/risk.pdf)

demand, economic shifts, government legislation and a multitude of conditions which are beyond the control of the decision maker. These are the type of decisions facing the senior executives of large corporations who commits huge resources often on gut feeling.

A situation of uncertainty arises when there are more than one possible consequences of selecting any course of action.

Decision making under uncertainty, as under risk, involves alternative actions whose payoffs depend on the states of nature. Specifically, the payoff matrix of a decision problem with m alternative actions and n states of nature can be represented by a m × n matrix, as follows;

	s_1	s_2	s_3	.....	s_n
a_1	$p(a_1, s_1)$	$p(a_1, s_2)$	$p(a_1, s_3)$		$p(a_1, s_n)$
a_2	$p(a_2, s_1)$	$p(a_2, s_2)$	$p(a_2, s_3)$		$p(a_2, s_n)$
a_3	$p(a_3, s_1)$	$p(a_3, s_2)$	$p(a_3, s_3)$		$p(a_3, s_n)$
.					
.					
.					
a_m	$p(a_m, s_1)$	$p(a_m, s_2)$	$p(a_m, s_3)$		$p(a_m, s_n)$

The element a_i represents action i and the element s_j represents state of nature j. The payoff or outcome associated with action ai and state s_j is $p(a_i, s_j)$. In decision making under uncertainty, the probability distribution associated with the states s_j, j = 1, 2, c, n, is unknown as it cannot be determined. This absence of information has led to the development of some special decision criteria which may be categorised as

1. Maximin (Minimax)
2. Laplace
3. Savage
4. Hurwicz

Each of the above are discussed briefly with illustration

1. The Minimax (Maximin) Criterion

The maximin (minimax) criterion is based on the conservative attitude of making the best of the worst-possible conditions). The logic is simple. The decision maker would zero upon such a decision which will give him optimum results under the given condition.

If $p(a_i, s_j)$ is loss or cost , then selection of an action is made on the basis of minimax criterion as the objective would be to minimise loss or cost (as the payoff denotes loss or cost)

On the contrary, If $p(a_i, s_j)$ is profit or revenue , then selection of an action is made on the basis of maximin criterion as the objective would be to maximise profit or revenue (as the payoff denotes profit or revenue).

2. The Laplace Criterion

This is based on the principle of insufficient reason. The simple argument is that because the probability distributions are not known, there is no reason to believe that the probabilities associated with the states of nature are different. The alternatives are thus evaluated on the basis of the assumption that all states of nature are

equally likely to occur. Given that the payoff $p(a_i, s_j)$ represents gain, the best alternative is the one that yields the maximum expected value (using equal probability). And in situation payoff $p(a_i, s_j)$ represents loss the minimum value represents the best alternative.

3. The Savage Criterion

Under this rule, the degree of conservatism in the minimax (maximin) is moderated by replacing the (gain or loss) payoff matrix $p(a_i, s_j)$ with a loss (or regret) matrix, $r(a_i, s_j)$. The following illustration is given as to why the transformation is suggested and how it is undertaken. This is also known as minimax regret criterion.

The following loss matrix is noted from a particular decision problem. The $p(a_i, s_j)$, where the payoff is loss, is given as

	s_1	s_2
a_1	₹ 11,000	₹ 90
a_2	₹ 10,000	₹ 10,000

Since the payoff matrix represents cost, Minimax criterion is to be applied on the basis of the conservative principle. Thus maximum values of each row is considered and the minimum of them is considered and the action representing the minimax value is the best decision.

	s_1	s_2	Row Max	
a_1	₹ 11,000	₹ 90	₹ 11,000	
a_2	₹ 10,000	₹ 10,000	₹ 10,000	← Minimax

The application of the minimax criterion shows that a_2, with a definite loss of ₹10,000, is the preferred alternative. However, it may be better to choose a_1 because there is a chance of limiting the loss to ₹90 given that s_2 occurs. This is the situation which is posited by the Savage rule. Transforming the above payoff matrix into a regret would be helpful to moderate the degree of conservatism. Transforming the above payoff into a regret matrix, the following is derived at. The regret is arrived at deducting the minimum value of a column from all the values of that column. This is on the basis of the opportunity cost principle.

	s_1	s_2	Row Max	
a_1	₹ 1,000	₹ 0	₹ 1,000	← Minimax
a_2	₹ 0	₹ 9,910	₹ 9,910	

4. The Hurwicz Criterion

The minimax and the maximin criteria, discussed above, assumes that the decision-maker is either optimistic or pessimistic. These are, as such, two extreme cases and a more realistic approach would be to consider the degree of optimism or pessimism of the decision-maker. The Hurwicz criterion, is designed to represent different decision-making attitudes, ranging from the most liberal (optimistic) to the most conservative (pessimistic). This is also referred as condition of equal likelihood.

One parameter α is used as the index of optimism. If $\alpha = 0$, then the criterion reduces to conservative minimax criterion, on the basis of the best of the worst conditions. If $\alpha = 1$, then the criterion is generous because it is based on the underlying assumption of the best of the best conditions. The degree of optimism (or pessimism) can be adjusted by selecting a value of a between 0 and 1. In the absence of strong feeling regarding extreme optimism and extreme pessimism, $\alpha = 0.5$ which indicates a fair choice, neither.

Illustration 8 (Illustration on Profit Matrix)

Given the following Payoff table of profits generated by an entity under differing condition of the future state of nature.

Alternatives	States of Nature			
	S1	S2	S3	S4
A1	3	5	8	-1
A2	6	5	2	0
A3	0	5	6	4

State which can be chosen as the best act using:

(a) Maximax,

(b) Maximin,

(c) Minimax regret (Savage criterion),

(d) Equal likelihood (Laplace criterion),

(e) Hurwicz Alpha criterion α=0.4

Solution:

a) Maximax:

Max (Max Ai) = Max (8, 6, 6) = 8

Decision:

Select A1

b) Maximin:

Max (Min Ai) =Max (-1, 0 ,0) =0

Decision: Select A2 or A3

c) Savage Criterion (Minimax regret)

i. Formulation of the regret table[17]

The Regret Table

Alternatives	States of Nature			
	S1	S2	S3	S4
A1	-3	0	0	-5
A2	0	0	-6	-4
A3	-6	0	-2	0

ii. The negative signs are dropped as the table represents loss (regret) matrix while the original matrix is a profit matrix.

[17] Subtracting the maximum (since the problem refers to minimax regret) of each column from each element of that particular column.

The Regret Table

Alternatives	States of Nature			
	S1	S2	S3	S4
A1	3	0	0	5
A2	0	0	6	4
A3	6	0	2	0

iii. Min (Max Ai) = Min (5, 6, 6) = 5

Decision: Select A1

d) Laplace: Under this condition, the associated probabilities are considered to be equal for each state of nature i.e., 1/4 (as there are four states of nature).

Expected pay-offs are :

Alternatives	States of Nature				Expected Value = Payoff × Probabilities $[EV= \sum P(X_i) \times X_i]$
	S1	S2	S3	S4	
	0.25[18]	0.25	0.25	0.25	
A1	3	5	8	-1	3.75
A2	6	5	2	0	3.25
A3	0	5	6	4	3.75

From the above calculation of expected value, it is noted that A1 and A3 has the maximum expected pay-off.

Decision: Select A1 or A2

e) The Hurwicz Criterion[19]

$D = \alpha$ (Maximum in A_i) + (1 – α) (Minimum in A_i) [$\alpha = 0.4$]

$D(A1) = (0.4 \times 8) + (0.6 \times -1) = 2.6$

$D(A2) = (0.4 \times 6) + (0.6 \times 0) = 2.4$

$D(A3) = (0.4 \times 6) + (0.6 \times 0) = 2.4$

D(A1) has the maximum Expected Value.

Decision: Select A1

[18] These are the assumed probabilities, which is the fundamental assumption in the Laplace criterion

[19] It is important to note that α is a parameter of optimism. If α = 0, then the criterion reduces to conservative minimax criterion which seeks the best of the worst conditions. If α = 1, then the criterion is generous because it seeks the best of the best conditions. The degree of optimism (or pessimism) can be adjusted by selecting a α value between 0 and 1. From this point of view, the formula for D (expected value) would reverse if the problem is profit maximisation or cost minimisation.

Illustration 9[20] (Illustration on Cost Matrix)

Julien Point School (JPS) is preparing a summer camp in the jungles of Bagora, District of Darjeeling to train individuals in wilderness survival. JPS estimates that attendance can fall into one of four categories: 200, 250, 300, and 350 persons. The cost of the camp will be the smallest when its size meets the demand exactly. Deviations above or below the ideal demand levels incur additional costs resulting from constructing more capacity than needed or losing income opportunities when the demand is not met. Letting a_1 to a_4 represent the sizes of the camp (200, 250, 300, and 350 persons) and s_1 to s_4 the level of attendance, the following table summarizes the cost matrix (in thousands of Rupees) for the situation:

	s_1	s_2	s_3	s_4
a_1	5	10	18	25
a_2	8	7	12	23
a_3	21	18	12	21
a_4	30	22	19	15

State the best alternative using: (i) Minimax, (ii) Laplace, (iii) Savage Criterion (Minimax Regret), (iv) Hurwicz Criterion.

Solution:

The problem is analyzed using the following:

i. The Minimax Criterion

	s_1	s_2	s_3	s_4	Row Max	
a_1	5	10	18	25	25	
a_2	8	7	12	23	23	
a_3	21	18	12	21	**21**	← Minimax
a_4	30	22	19	15	30	

ii. The Laplace Criterion

Assume equal probabilities (1/4) as there are four states of nature.

	s_1	s_2	s_3	s_4	EV= $\sum P(X_i) \times X_i$	Figures in ₹ thousand
a_1	5	10	18	25	1/4 (5+10+18+25)=14.5	₹ 14,500
a_2	8	7	12	23	1/4 (8+7+12+23) =12.5	**₹ 12,500**
a_3	21	18	12	21	1/4 (21+18+12+21) =18.0	₹ 18,000
a_4	30	22	19	15	1/4 (30+22+19+15) =21.5	₹ 21,500

Since it is a cost minimisation problem, decision a_2 would be selected which implicates the lowest cost of ₹12,500.

[20] Adopted from Chapter 15, Decision Analysis and Games (Operations Research - An Introduction, Tenth Edition, by Hamdy A. Taha)

iii. The Savage Criterion

This criterion posits the formulation of a regret matrix. The regret matrix is determined by subtracting 5, 7, 12, and 15 from columns 1 to 4, respectively, and so the following regret matrix is obtained.

	s_1	s_2	s_3	s_4	Row Max	
a_1	0	3	6	10	10	
a_2	3	0	0	8	**8**	← Minimax
a_3	16	11	0	6	16	
a_4	25	15	7	0	25	

iv.The Hurwicz Criterion[19]

The following table summarizes the computation

Alternative	Row Min	Row Max	α (Row Min) + (1-α) (Row Max)
a1	5	25	25 -20α[21]
a2	7	23	23 -16α
a3	12	21	21 - 9α
a4	15	30	30 -15α

The decision maker will have to decide upon the appropriate α, and thus he can decide upon the optimum alternative.

[21] α(5) + (1-α)(25) = α5+ 25 -25α = 25 -20α, and so forth (for the remaining values in the column).

Decision Tree

10.4

A decision tree shows a complete picture of a potential decision and allows a manager to graph alternative decision paths. Decision trees are a useful way to analyse hiring, marketing, investments, equipment purchases, pricing, and similar decisions that involve a progression of smaller decisions. Generally, decision trees are used to evaluate decisions under conditions of risk. Decision making is the core function of management. New tools for analysis that aid decision making are being developed. One such tool is the decision tree. It is essentially a visual graph that uses the branching method to map every possible outcome of a particular decision.

The term decision tree comes from the graphic appearance of the technique that starts with the initial decision shown as the base. The various alternatives, based upon possible future environmental conditions, and the payoffs associated with each of the decisions branch from the trunk.

Decision trees force a manager to be explicit in analysing conditions associated with future decisions and in determining the outcome of different alternatives. The decision tree is a flexible method. It can be used for many situations in which emphasis can be placed on sequential decisions, the probability of various conditions, or the highlighting of alternatives.

Decision trees are diagrams which illustrate the choices and possible outcomes of a decision. A decision tree is a pictorial method of showing a sequence of interrelated decisions and their expected outcomes. Decision trees can incorporate both the probabilities of, and values of, expected outcomes, and are used in decision-making.

More complex probability questions, although solvable using the basic principles, require a clear logical approach to ensure that all possible choices and outcomes of a decision are taken into consideration.

Decision trees are a useful means of interpreting such probability problems.

Merits of Decision Trees

- All the possible choices that can be made are shown as branches on the tree.
- All the possible outcomes of each choice are shown as subsidiary branches on the tree.

Constructing a decision tree

There are two stages in preparing a decision tree.

- Drawing the tree itself to show all the choices and outcomes
- Putting in the numbers (the probabilities, outcome values and EVs)

Every decision tree starts from a decision point with the decision options that are currently being considered.

(a) It helps to identify the decision point, and any subsequent decision points in the tree, with a symbol. Here, we shall use a square shape.

(b) There should be a line, or branch, for each option or alternative.

It is conventional to draw decision trees from left to right, and so a decision tree will start as follows:

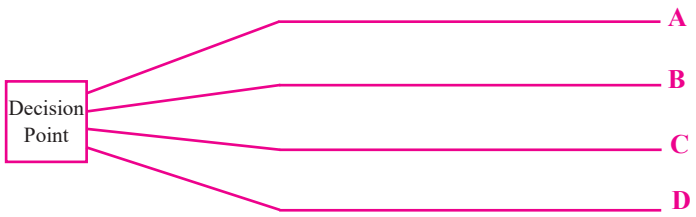

Figure 10.1 : Conventional decision tree

The square is the decision point, and A, B, C and D represents four alternatives from which a choice must be made (such as buy a new machine with cash, hire a machine, continue to use existing machine, raise a loan to buy a machine).

If the outcome from any choice is certain, the branch of the decision tree for that alternative is complete.

If the outcome of a particular choice is uncertain, the various possible outcomes must be shown.

We show the various possible outcomes on a decision tree by inserting an outcome point on the branch of the tree. Each possible outcome is then shown as a subsidiary branch, coming out from the outcome point. The probability of each outcome occurring should be written on to the branch of the tree which represents that outcome.

To distinguish decision points from outcome points, a circle will be used as the symbol for an outcome point.

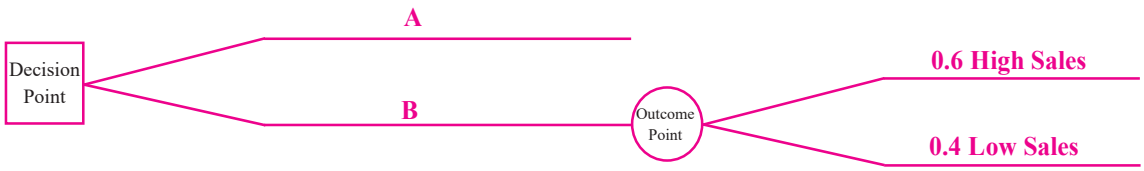

Figure 10.2 : Decision points and Outcome points

It is assumed that, there are two choices facing the decision-maker, A and B. The outcome if A is chosen is known with certainty, but if B is chosen, there are two possible outcomes, high sales (0.6 probability) or low sales (0.4 probability).

When several outcomes are possible, it is usually simpler to show two or more stages of outcome points on the decision tree.

Illustration 10

Several possible outcomes

A company can choose to launch a new product XYZ or not. If the product is launched, expected sales and expected unit costs might be as follows:

Sales		Unit costs	
Units	Probability	₹	Probability
10,000	0.8	6	0.7
15,000	0.2	8	0.3

a) The decision tree could be drawn as follows:

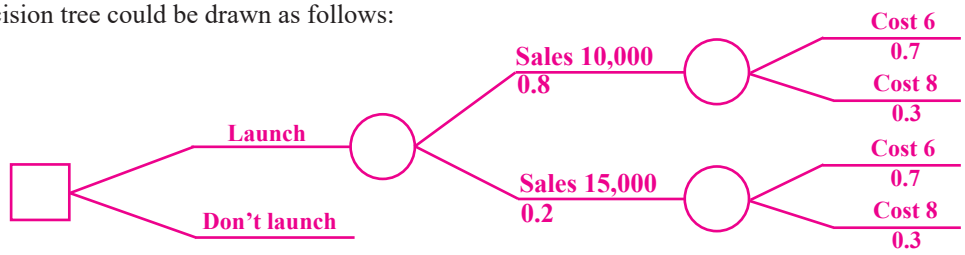

Figure 10.3 : Drawing of Decision tree

(b) The layout shown above will usually be easier to use than the alternative way of drawing the tree, which is as follows:

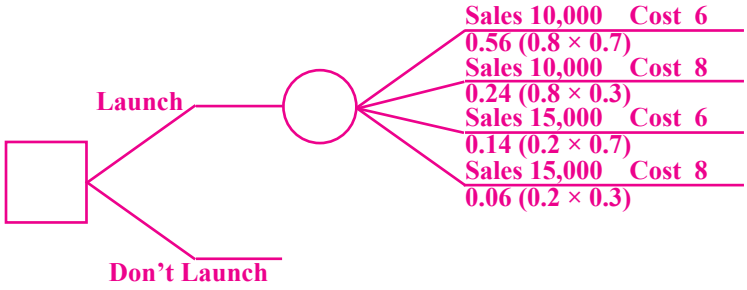

Figure 10.4 : Alternative way of drawing the Decision tree

Sometimes, a decision taken now will lead to other decisions to be taken in the future. When this situation arises, the decision tree can be drawn as a two-stage tree, as follows:

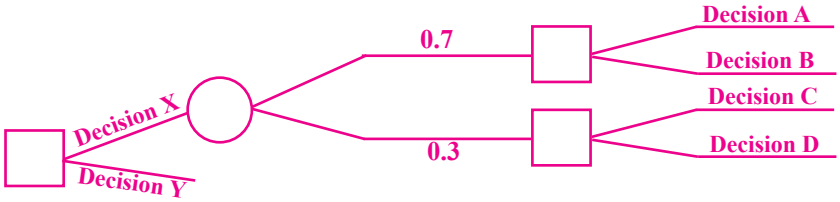

Figure 10.5 :Two-stage Decision tree

In this tree, either a choice between A and B or else a choice between C and D will be made, depending on the outcome which occurs after choosing X.

The decision tree should be in chronological order from left to right. When there are two-stage decision trees, the first decision in time should be drawn on the left.

Illustration 11 (Example of Decision Tree)

LT Ltd. owns land in Bangalore and intends to build a project development on the site. The company is deciding on whether to build a small, medium or large development. Demand is uncertain and fluctuates; demand could be low, medium or high. Management at LT Ltd. has determined profit payoffs will be:

(all amounts in ₹ 000s)

Alternatives		Demand Low	Medium	High
Small	d1	1,400	1,400	1,400
Medium	d2	1,100	1,600	1,600
Large	d3	(1,300)	1,200	2,100

Management has determined the probabilities of demand to be:

Low = P (low) = .20

Medium = P (medium) = .35

High = P (high) = .45

Then Decision Tree Approach can be applied as under:

Solution:

Decision Tree Approach

Payoff table in decision tree format

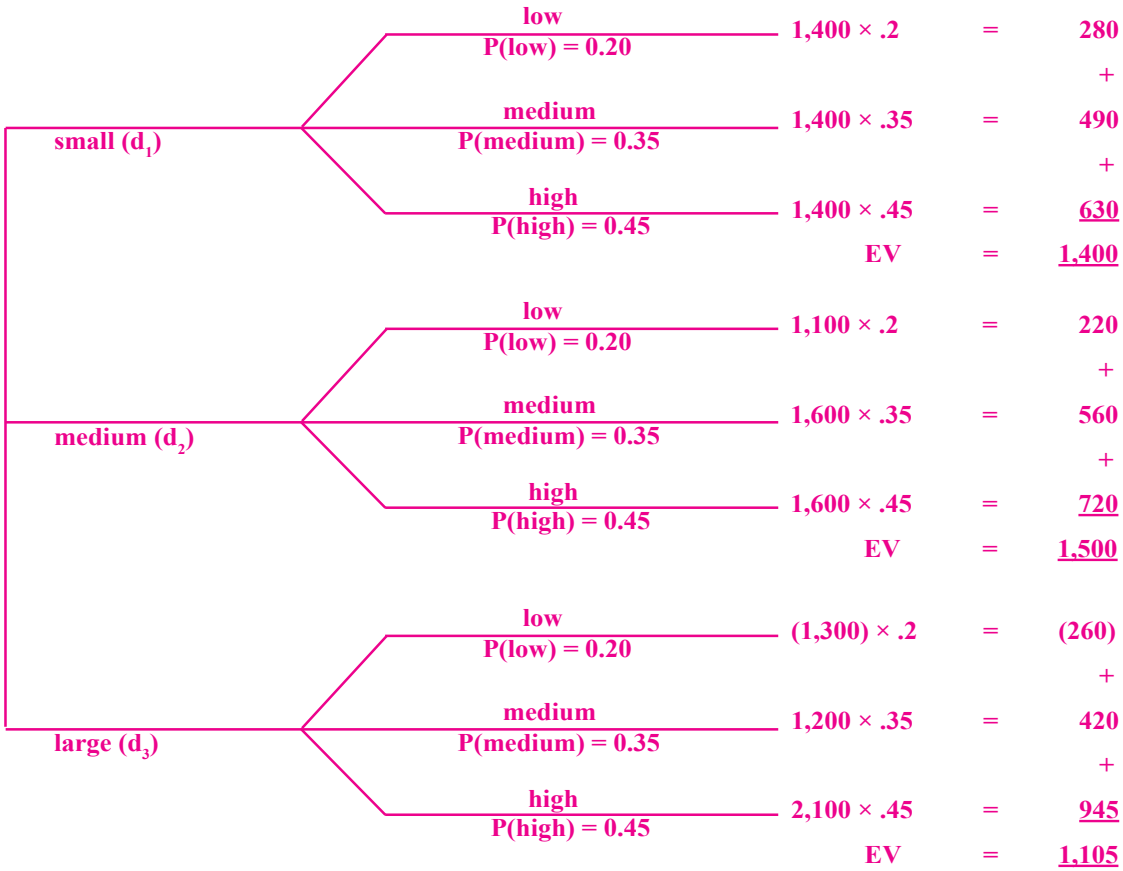

Figure 10.6 : Payoff table in Decision tree format

Conclusion: Build a medium complex as this alternative provides the highest expected value

Illustration 12

B Ltd. has a new wonder product, the V, of which it expects great things. At the moment the company has two courses of action open to it, to test market the product or abandon it.

If the company test markets it, the cost will be ₹ 1,00,000 and the market response could be positive or negative with probabilities of 0.60 and 0.40.

If the response is positive the company could either abandon the product or market it full scale.

If it markets the V in full scale, the outcome might be low, medium or high demand, and the respective net gains/ (losses) would be (200), 200 or 1,000 in units of ₹1,000 (the result could range from a net loss of ₹ 2,00,000 to a gain of ₹10,00,000). These outcomes have probabilities of 0.20, 0.50 and 0.30 respectively.

If the result of the test marketing is negative and the company goes ahead and markets the product, estimated losses would be ₹ 6,00,000.

If, at any point, the company abandons the product, there would be a net gain of ₹ 50,000 from the sale of scrap. All the financial values have been discounted to the present.

Required

(a) Draw a decision tree.

(b) Include figures for cost, loss or profit on the appropriate branches of the tree.

Solution:

The starting point for the tree is to establish what decision has to be made now. What are the options?

(a) To test market

(b) To abandon

The outcome of the 'abandon' option is known with certainty. There are two possible outcomes of the option to test market, positive response and negative response.

Depending on the outcome of the test marketing, another decision will then be made, to abandon the product or to go ahead.

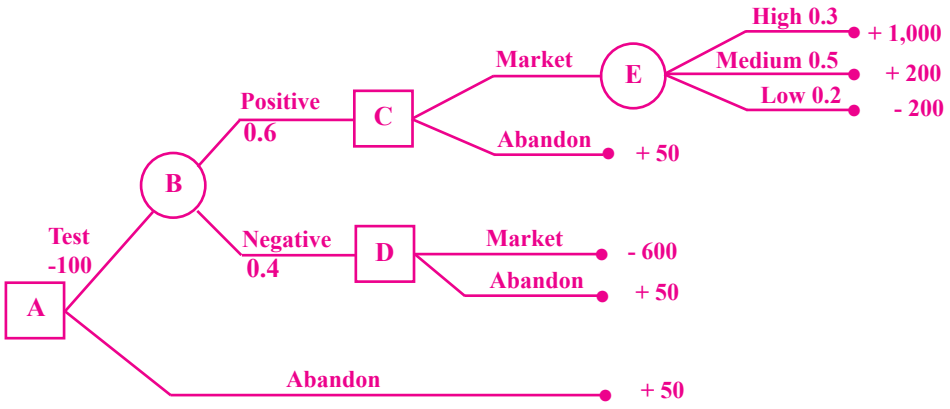

Figure 10.7 : Decision Making through Decision Tree Approach

Evaluating decisions by using decision trees has a number of limitations as follows:

(a) The time value of money may not be taken into account.

(b) Decision trees are not very suitable for use in complex situations.

(c) The outcome with the highest EV may have the greatest risks attached to it. Managers may be reluctant to take risks which may lead to losses.

(d) The probabilities associated with different branches of the 'tree' are likely to be estimates, and possibly unreliable or inaccurate.

Illustration 13 (Problem on Expected Value)

TT Newsagents stocks a weekly health magazine. The owner buys the magazines for ₹ 0.30 each and sells them at the retail price of ₹0.50 each.

At the end of the week unsold magazines are obsolete and have no value. The estimated probability distribution for weekly demand is shown below.

Weekly demand in units	Probability
20	0.20
30	0.55
40	0.25
	1.00

You are required to calculate the following:

(i) What is the expected value of demand?

(ii) If the owner is to order a fixed quantity of magazines per week how many should that be?

Assume no seasonal variations in demand.

Solution

EV of demand (units per week) = (20 × 0.20) + (30 × 0.55) + (40 × 0.25) = 30.5 units per week

The next step is to set up a decision matrix of possible strategies (numbers bought) and possible demand.

The 'pay-off' from each combination of action and outcome is then computed.

No sale = Cost of ₹ 0.30 per magazine

Sale = Profit of ₹ 0.20 per magazine (₹0.50 - ₹ 0.30)

Probability	Outcome (Numbers demanded)	Decision (Profit) (Numbers bought)		
		20	30	40
		₹	₹	₹
0.20	20	4.00	1.00*	(2.00)
0.55	30	4.00	6.00	3.00
0.25	40	4.00	6.00	8.00
	EV	4.00	5.00**	3.25

* Buy 30 and sell only 20 gives a profit of (20 × ₹0.5) – (30 × ₹0.3) = ₹1

** (0.2 ×1) + (0.55 × 6) + (0.25 × 6) = 5

The strategy which gives the highest expected pay-off is to stock 30 magazines each week.

Conclusion:

Probability is a numerical measurement of uncertainty. When a probability is based on counting and observed frequencies, it is objective. When a probability is an expression of whether an event in business will or will not occur, it may be based on the relative frequency of similar events having occurred in the past, or it may be based on someone's judgment. Either way, the determination of probability has strong subjective elements.

Therefore, the concept of probability as it is used in business is a numerical measure of the belief of an individual in the occurrence or non-occurrence of an event. The probability assigned to an event depends upon the information and knowledge that the decision-maker has and uses in assessing the probability. As such, probability assessment is clearly subjective, individual, and dependent upon information. In fact, it has been said that probability does not exist in any absolute or objective sense.

Thus, these statistical methods of dealing with risk and uncertainty are only means of obtaining a recommended decision alternative or an optimal strategy for the purpose of planning, budgeting, and decision-making. The actual results from the implementation of the decision will probably be quite different from the calculated expected value. The decision-maker's judgment is the deciding factor.

Illustration 14 (Problem on Probabilistic Budget)

The Profit Budget of ABC Company is given below:

Profit Budget for year ending 31st March 2022

		₹
Sales (1,00,000 units @ ₹10)		10,00,000
Variable costs:		
Manufacturing (₹ 5 per unit)	5,00,000	
Marketing (₹0.50 per unit)	50,000	5,50,000
Contribution:		4,50,000
Fixed costs:		
Manufacturing	2,00,000	
Marketing	50,000	
Administrative	1,00,000	3,50,000
Profit Before Tax		1,00,000
Tax (assumed at 50%)		50,000
Profit after Tax		50,000

Note: Manufacturing variable cost is:

₹ 5.10 per unit at volume of 80,000 units

₹ 5.00 per unit at volume of 1,00,000 units

₹ 4.80 per unit at volume of 1,10,000 units

The marketing manager has given the sales forecast as follows.

Probability 0.3 - 80,000 units

Probability 0.5 - 1,00,000 units

Probability 0.2 - 1,10,000 units

The production manager has indicated the variable manufacturing cost to be as follows:

Probability 0.2 - ₹ 5.10 per unit

Probability 0.6 - ₹ 5.00 per units

Probability 0.2 - ₹ 4.80 per units

The Management Accountant is to work out the profit Budget taking the above factors into account. All other costs are as given earlier.

Prepare a Probabilistic Budget.

Solution:

In the table below, a three-way forecast is given:

Description	**Pessimistic**	**Most Likely**	**Optimistic**
	₹	₹	₹
Sales	8,00,000	10,00,000	11,00,000
Variable Costs:			
Manufacturing	4,08,000	5,00,000	5,28,000
Marketing	40,000	50,000	55,000
Contribution	3,52,000	4,50,000	5,17,000
Fixed Costs:			
Manufacturing	2,00,000	2,00,000	2,00,000
Marketing	50,000	50,000	50,000
Administration	1,00,000	1,00,000	1,00,000
Profit Before Tax	2,000	1,00,000	1,67,000
Tax (assumed 50%)	1,000	50,000	83,500
Profit after Tax	1,000	50,000	83,500

The Probabilistic Profit Budget is shown below:

Budget

Volume	Variable Cost per unit	PAT	Joint probability (JP)	PAT×JP
80,000				
P = 0.3	5.10(P=0.2)	1,000	0.06	60
	5.00 (P=0.6)	5,000	0.18	900
	4.80 (P=0.2)	13,000	0.06	780
1,00,000				
P = 0.5	5.10 (P=0.2)	45,000	0.10	4,500
	5.00 (P=0.6)	50,000	0.30	15,000
	4.80(P=0.2)	60,000	0.10	6,000
1,10,000				
P = 0.2	5.10 (P=0.2)	67,000	0.04	2,680
	5.00 (P=0.6)	72,500	0.12	8,700
	4.80 (P=0.2)	83,500	0.04	3,340
EV				41,960

Thus, it can be observed that a realistic profit estimate will be ₹ 41,960 and not ₹ 50,000.

Illustration 15 (Statistical Measurements of Cash Flow Variability)

When forecasting cash flows for investment projects, we might make several sets of forecasts for each project to reflect the various alternative states of the economy that might ensue. If we are comparing two project proposals, both for one-year projects, we might make several forecasts for the cash flows, as follows:

	Project X	Project Y
	₹	₹
Economy in a deep recession	2,00,000	1,00,000
Economy in a mild recession	2,50,000	2,00,000
Economy stable	3,00,000	3,00,000
Economy in a minor expansion	3,50,000	4,00,000

Suppose economists forecast that the probability of a deep recession occurring next year is 5%, a mild recession

is 10%, a stable economy is 50%, a minor expansion is 25%, and a major expansion is 10%. Using these projections, we can calculate the expected value of the cash flows for both projects:

	Probability	Project X		Project Y	
	(P)	Cash Flow	CF x P	Cash Flow	CF x P
		₹	₹	₹	₹
Economy in a deep recession	5%	2,00,000	10,000	1,00,000	5,000
Economy in a mild recession	10%	2,50,000	25,000	2,00,000	20,000
Economy stable	50%	3,00,000	1,50,000	3,00,000	1,50,000
Economy in a minor expansion	25%	3,50,000	87,500	4,00,000	1,00,000
Economy in a major expansion	10%	4,00,000	40,000	5,00,000	50,000
Expected Value			**3,12,500**		**3,25,000**

The expected value of the cash flows for each of the two projects is simply a weighted average of the possible cash flows, with the weights being the probabilities of each occurrence. The expected value of Project Y's cash flows is higher than the expected value of Project X's cash flows.

The expected value is the average value, or mean, of the possible values. According to the data above, if the same cash flow could be repeated over and over again, 5% of the time the cash flow would be ₹ 2,00,000, 10% of the time it would be ₹ 2,50,000, and so forth. The weighted average of these potential cash flows is the expected value.

The problem with using expected value as a forecast for a specific project is that any given project has only one opportunity to achieve its cash flow for its duration and then the project is complete. The cash flow actually achieved for Project X could be anywhere from ₹ 2,00,000 to ₹ 4,00,000. Once one of the possible cash flows has been achieved, we will know that the probability of that cash flow occurring was 100% while the probability of the other cash flows occurring was zero.

An expected value is a "long-run" average value for a random variable. As a result, an expected value is more reliable as a long-run average forecast and less reliable as a forecast for the net cash flow for an individual project at any given moment in time.

Despite not being a reliable forecast, expected value is often used to project future cash flow from individual projects because it is the best method available for obtaining a forecast.

Illustration 16

A manager is considering whether to make product X or product Y, but only one can be produced. The estimated sales demand for each product is uncertain. A detailed investigation of the possible sales demand for each product gives the following probability distribution of the profits for each product.

Product X probability distribution

Outcome	Estimated Probability	Weighted Profit (₹)
Profits of ₹6,00,000	0.10	60,000
Profits of ₹7,00,000	0.20	1,40,000
Profits of ₹8,00,000	0.40	3,20,000
Profits of ₹9,00,000	0.20	1,80,000
Profits of ₹10,00,000	0.10	1,00,000
Expected value		**8,00,000**

Product Y probability distribution

Outcome	Estimated probability	Weighted Profit (₹)
Profits of ₹ 4,00,000	0.05	20,000
Profits of ₹ 6,00,000	0.10	60,000
Profits of ₹ 8,00,000	0.40	3,20,000
Profits of ₹ 10,00,000	0.25	2,50,000
Profits of ₹12,00,000	0.20	2,40,000
Expected value		**8,90,000**

Based on the information given, assumed to be perfect, the manger should make Product X, having higher expected value.

Since decision problems exist in an uncertain environment, it is necessary to consider those uncontrollable factors that are outside the decision-maker's control and that may occur for alternative courses of action. These uncontrollable factors are called events or states of nature. For example, in a product launch situation, possible states of nature could consist of events such as a similar product being launched by a competitor at a lower price, at the same price, at a higher price or no similar product being launched at all.

The likelihood that an event or state of nature will occur is known as its probability, and this is normally expressed in decimal form with a value between 0 and 1. A value of 0 denotes a nil likelihood of occurrence, whereas a value of 1 signifies absolute certainty – a definite occurrence. A probability of 0.4 means that the event is expected to occur four times out of ten. The total of the probabilities for events that can possibly occur must sum to 1.0.

For example, if an examiner indicates that the probability of a student passing an examination is 0.7 then this means that the student has a 60 per cent chance of passing the examination. Given that the pass/fail alternatives represent an exhaustive listing of all possible outcomes of the event, the probability of not passing the examination is 0.4.

The information can be presented in a probability distribution. A probability distribution is a list of all possible

outcomes for an event and the probability that each will occur. The probability distribution for the above example is as follows:

Outcome	Probability
Pass the examination	0.6
Do not pass examination	0.4
Total	1.0

Some probabilities are known as objective probabilities because they can be established mathematically or compiled from historical data. Tossing a coin and throwing a dice are examples of objective probabilities.

For example, the probability of heads occurring when tossing a coin logically must be 0.5. This can be proved by tossing the coin many times and observing the results. Similarly, the probability of obtaining number 1 when a dice is thrown is 0.166 (i.e. one-sixth). This again can be ascertained from logical reasoning or recording the results obtained from repeated throws of the dice.

It is unlikely that objective probabilities can be established for business decisions, since many past observations or repeated experiments for particular decisions are not possible; the probabilities will have to be estimated based on managerial judgement. Probabilities established in this way are known as subjective probabilities because no two individuals will necessarily assign the same probabilities to a particular outcome. Subjective probabilities are based on an individual's expert knowledge, past experience and observations of current variables that are likely to have an impact on future events. Such probabilities are unlikely to be estimated correctly, but any estimate of a future uncertain event is bound to be subject to error.

The advantage of this approach is that it provides more meaningful information than stating the most likely outcome.

EXERCISE

Theoretical Questions

Multiple Choice Question

1. A type of decision-making environment is
   - A Certainty
   - B. Uncertainty
   - C. Risk
   - D. All of these
2. Which of the following criterion is not used for decision-making under uncertainty?
   - A. Maximin
   - B. Maximax
   - C. Minimax
   - D. Minimize expected loss
3. Decision theory is concerned with
   - A. Methods of arriving at an optimal decision
   - B. Selecting optimal decision in a sequential manner
   - C. Analysis of information that is available
   - D. All of these
4. Which of the following criterion is not applicable to decision-making under risk?
   - A. Maximize expected return
   - B. Maximize return
   - C. Minimize expect regret
   - D. Knowledge of likelihood occurrence of each state of nature
5. The minimum expected opportunity loss (EOL) is
   - A. Equal to EVPI
   - B. Minimum regret
   - C. Equal to EMV
   - D. Both (A) and (B)
6. The expected value of perfect information (EVPI) is
   - A. Equal to expected regret of the optimal decision under risk

B. The utility of additional information

C. Maximum expected opportunity loss

D. None of the above

7. The value of the coefficient of optimism (a) is needed while using the criterion of

A. Equally likely

B. Maximin

C. Realism

D. Minimax

8. The decision-maker's knowledge and experience may influence the decision-making process wi.en using the criterion of

A. Maximax

B. Maximax regret

C. Realism

D. Maximin

9. The difference between the expected profit under conditions of risk and the expected profit with perfect information is called

A. The expected value of perfect information

B. Expected marginal loss

C. None of the above

D. Any one of the above

10. A situation in which a decision maker knows all of the possible outcomes of a decision and also knows the probability associated with each outcome is referred to as

A. Certainty.

B. Risk.

C. Uncertainty.

D. Strategy.

11. Which of the following methods of selecting a strategy is consistent with risk averting behaviour?

A. If two strategies have the same expected profit, select the one with the smaller standard deviation.

B. If two strategies have the same standard deviation, select the one with the smaller expected profit.

C. Select the strategy with the larger coefficient of variation.

D. All of the above are correct.

12. Which one of the following does not measure risk?

    A. Coefficient of variation

    B. Standard deviation

    C. LPP

    D. All of the above are measures of risk.

13. A situation in which a decision maker must choose between strategies that have more than one possible outcome when the probability of each outcome is unknown is referred to as

    A. Diversification.

    B.. Certainty.

    C. Risk.

    D. Uncertainty.

14. If a decision maker is risk averse, then the best strategy to select is the one that yields the

    A. Highest expected payoff.

    B. Lowest coefficient of variation.

    C. Highest expected utility.

    D. Lowest standard deviation.

15. Circumstances that influence the profitability of a decision are referred to as

    A. Strategies.

    B. A payoff matrix.

    C. States of nature.

    D. the marginal utility of money.

16. A strategy that yields an expected monetary payoff of zero is called a

    A. Risk-neutral strategy.

    B. Fair game.

    C. Zero-sum game.

    D. Certainty equivalent

17. A matrix that, for each state of nature and strategy, shows the difference between a strategy's payoff and the best strategy's payoff is called

    A. A maximin matrix.

    B. A minimax regret matrix.

    C. A payoff matrix.

    D. An expected utility matrix.

18. The sequence of possible managerial decisions and their expected outcome under each set of circumstances can be represented and analysed by using

    A. The minimax regret criterion.

    B. A decision tree.

    C. A payoff matrix.

    D. Simulation.

19. The expected value of perfect information is calculated by subtracting:

    A. The minimum expected opportunity loss from the expected opportunity loss with perfect information.

    B. The maximum EMV from the minimum expected opportunity loss.

    C. EVSI from the expected return with perfect information.

    D. The maximum EMV from the expected return with perfect information.

20. The maximin criterion is a feature of which of the following?

    A. Deterministic model

    B. Decision-making under uncertainty

    C. Optimization

    D. Decision-making under certainty

Answer:

1- D, 2-D, 3-D, 4-B, 5-D, 6- A, 7- C, 8- C, 9- A, 10- B, 11- A, 12- C, 13- D, 14- C, 15-C, 16-B, 17-B, 18-B, 19-A, 20- B.

State True or False

1. Decision theory provides a method for rational decision making when the consequences are not fully known.
2. Companies benefit most from considering their risks when they are performing well and when markets are growing in order to sustain growth and profitability.
3. A decision maker is risk neutral if he is concerned with what will be the most likely outcome.
4. The decision outcome resulting from the same information may vary from manager to manager as a result of their individual attitude to risk.
5. "Risk" can be defined in many ways. One definition has a negative connotation: "a condition in which there is a possibility of an adverse deviation from a desired outcome."
6. The variance and standard deviation both give an idea of the variability of the possible values about the mean.

7. Standard deviation is always expressed in the same units as the distribution.
8. Uncertainty is risk that can be measured.
9. If the occurrence or non-occurrence of one event does not change the probability of the occurrence of the other event, the two events are said to be independent.
10. The expected value of an action is found by multiplying the probability of each potential outcome by its payoff.

Answer:

1- True, 2- True, 3- True, 4- True, 5- True, 6- True, 7- True, 8-False, 9- True, 10-True.

⊙ Fill in the Blanks

1. ……is a term that means a weighted average of the possible values using the probabilities as the weights.
2. A ……. amount of future cash flow is often thought of as an absolute number.
3. Approaches have been developed to choose the best option when the decision maker has several …….and there is uncertainty with respect to future events.
4. If a decision maker can estimate the ……. of the future events, these should be incorporated into the decision model.
5. …….is knowledge about the future that would enable us to make the best choice today for any possible situation in the future.
6. …….is the systematic process of gathering, analysing and reporting data about markets to investigate, describe measure, understand or explain a situation or problem facing a company or organisation.
7. ……. can be either primary (collected at first hand from a sample of respondents), or secondary (collected from previous surveys, other published facts and opinions, or from experts).
8. …….data tells us why consumers think/buy or act the way they do.
9. ……tables identify and record all possible outcomes (or pay-offs) in situations where the action taken affects the outcomes.
10. The ……decision rule suggests that a decision maker should select the alternative that offers the least unattractive worst outcome.

Answer:

1- Expected value, 2- Budgeted, 3- alternatives, 4- Probabilities, 5- Perfect information, 6- Market research, 7- Data, 8- Qualitative, 9- Pay-off, 10- Maximin.

⊙ Short Essay Type Questions

1. Which are the risks faced by a company?
2. What are the determinants of the risks that a company faces?

3. What do you mean by Expected Value?
4. What is Uncertainty and Risk?
5. How a Decision Tree is constructed?
6. Briefly explain the concepts 'Perfect Information' and 'Value of Perfect Information.'?
7. What is risk in decision making?
8. What is the Difference between Risk and Uncertainty?

⊙ Essay Type Questions

1. What is Decision Theory and how is it related to other Theories?
2. What are the most important methods to quantify risk?
3. Outline important characteristics of the risk evaluation process
4. Discuss the goal in the decision-making process.
5. What do you mean by uncertainty in decision making?
6. Explain why it can be helpful to involve others when making decisions involving uncertainty.
7. What is a Decision Tree and what are its applications?
8. What do you mean by Expected Value and Perfect Information?
9. What are the examples of risk and uncertainty?

Practical Problems

⊙ Multiple Choice Question

1. A company is choosing which of three new products to make (A, B or C) and has calculated likely payoffs under three possible scenarios (I, II or III), giving the following payoff table.

Profit (Loss) Scenario	Product Chosen		
	A	B	C
I	20	80	10
II	40	70	100
III	50	(10)	40

Using maximax, which product would be chosen?

A. Product A

B. Product B

C. Product C

D. None of the Products

2. ABC Co is trying to set the sales price for one of its products. Three prices are under consideration, and expected sales volumes and costs are as follows.

Price per unit	₹4	₹4.30	₹4.40
Expected sales volume (units)			
Best possible	16,000	14,000	12,500
Most likely	14,000	12,500	12,000
Worst possible	10,000	8,000	6,000

Fixed costs are ₹20,000 and variable costs of sales are ₹2 per unit.

Which price should be chosen?

A. ₹ 4

B. ₹4.30

C. ₹4.40

D. Insufficient data

3. Suppose a manager has to choose between mutually exclusive options A and B, and the probable outcomes of each option are as follows.

Option A		Option B	
Probability	Profit (₹)	Probability	Profit (₹)
0.8	5,000	0.1	(2,000)
0.2	6,000	0.2	5,000
$ $		0.6	7,000
		1.1	8,000

The expected value (EV) of profit will be:

A. ₹ 6,000

B. ₹ 4,500

C. ₹ 5,800

D. None of the above

4. ABC stocks a weekly lifestyle magazine. The owner buys the magazines for ₹0.30 each and sells them at the retail price of ₹0.50 each.

At the end of the week unsold magazines are obsolete and have no value. The estimated probability distribution for weekly demand is shown below.

Weekly demand in units	Probability
20	0.20
30	0.55
40	0.25
	1.00

What is the expected value of demand?

A. 30

B. 20

C. 25

D. None of the above

5. A manager has to choose between mutually exclusive options C and D and the probable outcomes of each option are as follows.

Options C		Options D	
Probability	Cost	Probability	Cost
	₹		₹
0.29	15,000	0.03	14,000
0.54	20,000	0.30	17,000
0.17	30,000	0.35	21,000
		0.32	24,000

Both options will produce an income of ₹30,000. Which should be chosen?

A. Option C

B. Option D

C. No Option

D. Both the Options

6. Suppose a businessman is trying to decide which of three mutually exclusive projects to undertake. Each of the projects could lead to varying net profit under three possible scenarios.

	Profits Project		
	D	E	F
Scenarios			
I	100	80	60
II	90	120	85
III	(20)	10	85

Which Project is to be selected?

A. D

B. E

C. F

D. Insufficient data

7. A company is considering which one of three alternative courses of action, A, B and C to take. The profit or loss from each choice depends on which one of four economic circumstances, I, II, III or IV will apply. The possible profits and losses, in thousands of pounds, are given in the following payoff table. Losses are shown as negative figures.

		Action		
		A	B	C
Circumstance	I	70	60	70
	II	–10	20	–5
	III	80	0	50
	IV	60	100	115

Required:

State which action would be selected using each of the maximax and maximin criteria?

A. A

B. B

C. C

D. All of the above

8. Suppose that a manager is trying to decide which of three mutually exclusive projects to undertake. Each of the projects could lead to varying net profits which are classified as outcomes I, II and III. The manager has constructed the following payoff table or matrix (a conditional profit table):

	Net profit if outcome turns out to be		
Project	I	II	III
A	₹50,000	₹65,000	₹80,000
B	₹70,000	₹60,000	₹75,000
C	₹90,000	₹80,000	₹55,000
Probability	0.2	0.6	0.2

Required:

Which project would be chosen using EV?

A. A

B. B

C. C

D. None of the above

9. A manager is trying to decide which of three mutually exclusive projects to undertake. Each of the projects could lead to varying net costs which the manager calls outcomes I, II and III. The following payoff table or matrix has been constructed:

		Outcomes (not profit)		
		I (Worst)	II (Most likely)	III (Best)
Project	A	50	85	130
	B	70	75	140
	C	90	100	110

Which project should be undertaken under minimax regret rules?

A. A

B. B

C. C

D. Insufficient data

10. The management of LT Company must choose whether to go ahead with either of two mutually exclusive projects: A or B. The expected profits are as follows:

	Profit if there is strong demand	Profit/(loss) if there is weak demand
Option A	₹4,000	₹(1,000)
Option B	₹1,500	₹ 500
Probability of demand	0.3	0.7

Ascertain what the decision would be, based on expected values, if no information about demand were available.

A. A

B. B

C. A and B

D. None

Answer:

1- C, 2-A, 3-C, 4-A, 5-A, 6- C, 7- B, 8- C, 9- B, 10- B.

⊙ Comprehensive Numerical Questions

1. ABC Corporation is introducing a new product and must decide on a selling price. The variable cost per unit is ₹5.60. Senior management has narrowed the pricing alternatives to two choices:

   ₹14.50 or ₹ 8.20 per unit. Management estimates sales levels and the probability of attaining these levels as:

   Selling price of ₹ 8.20:

Units	Probability
15,000	5%
10,000	85%
8,000	10%

   Selling price of ₹ 14.50:

Units	Probability
4,700	5%
3,800	65%
2,600	30%

As the volume levels noted above are in the relevant range, fixed costs remain constant regardless of the selling price chosen.

Required:

1. Using a payoff table, calculate the optimal price ABC Corporation should charge.
2. Restate Part 1 using a decision tree.

2. A dealer of perishable product earns a Profit of ₹3 per kg, if he can sell within two days, but incurs a loss of ₹2 per kg, if fails to do so. The estimated demand for the product and the relative probabilities are as given below:

Estimated Demand	Probability
0 kg	5%
1 kg	20%
2 Kg	40%
3 kg	25%
4 kg	10%

In order to maximize his profit, what should be the quantity of stock that he should hold?

3. XYZ Co. is considering rearranging its plant to increase efficiency. If the rearrangement is completely successful, anticipated operating costs will be ₹ 200,000 per annum. If the rearrangement is partially successful, anticipated operating costs are expected to be ₹ 310,000. If unsuccessful, operating costs are anticipated to be ₹ 510,000. The probability of complete success is 50%, partial success 30% and failure 20%. If the company does not rearrange, operating costs will be ₹ 400,000.

Required

1. Prepare a payoff table (alternatives are rearrange or do not rearrange).
2. Restate requirement 1 in a decision tree format.
3. XYZ has an opportunity to hire a consultant who could predict the success rate with certainty. How much should XYZ Co. be willing to pay for such a report?

4. You own the rights to sell Rolls at the local park. The weather network predicts the probability of rain at 60% for the game coming up tomorrow. You have to decide today whether you will set up Roll stand inside or outside. If you set up outside and it does not rain, you expect to sell 800 hot dogs, but if it rains you will only sell 200 Rolls. If you set up inside and it does not rain, you will sell 300 hot dogs, but, if it rains, you will sell 700 Rolls. Each Roll generates a contribution margin of ₹2.

Required

a. Where should you set up, inside or outside?

b. What is the most you would pay someone to predict (with 100% accuracy) tomorrow's weather?

c. At what rain probability would you be indifferent between setting up inside or outside?

5. Bajaj Company is considering three alternative machines to produce a new product. The cost structures (unit variable costs plus avoidable fixed costs) for the three machines are shown below. The selling price is unaffected by the machine used.

   Single purpose machine ₹ 0.60 + ₹ 20,000

   Semi-automatic machine ₹ 0.40 + ₹ 50,000

   Automatic machine ₹ 0.20 + ₹ 120,000

   The demand for units of the new product is described by the following probability distribution:

Demand	Probability
200,000	0.4
300,000	0.3
400,000	0.2
500,000	0.1

   Required

   Calculate expected demand. Calculate the expected costs of using the semi-automatic machine.

   Which machine should be selected?

6. Your client, Alpha Ltd., wants your advice on which of two alternatives he should choose. One alternative is to sell an investment now for ₹ 10,000. Another alternative is to hold the investment three days; after which he can sell it for a certain selling price based on the following probabilities:

Selling Price (₹)	Probability
5,000	0.4
8,000	0.2
12,000	0.3
30,000	0.1

   Required

   Would you recommend selling the investment now or hold the investment for three days?

7. The TTC Company is considering hiring several new employees to handle an overload from a new contract. If the new people are not hired, there will be delays in the contract work. The following payoff matrix has been prepared for analysing whether new people are needed:

	Hire New New People	Do Not Hire People
Retain new customers	₹100,000	₹75,000
Lose new customers	₹25,000	₹50,000

   Based on past experience, the company expects to retain 75% of the new customers with no new hires.

Required

Calculate the expected profit for the "no hire" decision.

8. Cement Co is a company specialising in the manufacture of cement, a product used in the building industry. The company has found that when weather conditions are good, the demand for cement increases since more building work is able to take place. Cement Co is now trying to work out the level of cement production for the coming year in order to maximise profits. The company has received the following estimates about the probable weather conditions and corresponding demand levels for the coming year:

Weather	Probability	Demand
Good	25%	350,000 bags
Average	45%	280,000 bags
Poor	30%	200,000 bags

Each bag of cement sells for ₹ 9 and costs ₹ 4 to make. If cement is unsold at the end of the year, it has to be disposed of at a cost of ₹ 0·50 per bag. Cement Co has decided to produce at one of the three levels of production to match forecast demand. It now has to decide which level of cement production to select.

Required

Construct a pay-off table to show all the possible profit outcomes.

⊙ Unsolved Case

1. Suppose you are the Manager Cost of your company and you have to choose between mutually exclusive options A and B. The probable outcomes of each option are as follows.

Option A		Option B	
Probability	Profit	Probability	Profit
0.8	5,000	0.1	(2,000)
0.2	6,000	0.2	5,000

Explain how you can arrive at a decision, for selecting either Option A or Option B?

Key Terms

Conservatism : Conservatism is associated with risk aversion and prudence.

Coefficient of variation: A ratio measure of dispersion derived by dividing the standard deviation by the expected value.

Decision tree: A diagram showing several possible courses of action and possible events and the potential outcomes for each of them.

Decision rule: A decision rule is a function which maps an observation to an appropriate action.

Decision Maker : A decision maker is risk neutral if he is concerned with what will be the most likely outcome.

Events: In the context of risk and uncertainty, factors that are outside the decision-maker's control, also known as states of nature.

Expected value: A figure calculated by weighting each of the possible outcomes by its associated probability.

Expected value of perfect information: The maximum amount it is worth paying for additional information in an uncertain situation, calculated by comparing the expected value of a decision if the information is acquired against the expected value in the absence of the information.

Independent Events : If the occurrence or non-occurrence of one event does not change the probability of the occurrence of the other event, the two events are said to be independent.

Joint Probability: A joint probability, in probability theory, refers to the probability that two events will both occur. In other words, joint probability is the likelihood of two events occurring together.

Maximax criterion : A decision rule based on the assumption that the best possible outcome will always occur and the decision-maker should therefore select the largest payoff.

Maximin criterion: A decision rule based on the assumption that the worst possible outcome will always occur and the decision-maker should therefore select the largest payoff under this assumption.

Mutually Exclusive Events : Mutually exclusive events are things that can't happen at the same time.

Objective probabilities: Probabilities that can be established mathematically or compiled from historical data.

Pay-off : A Payoff Table is a listing of all possible combinations of decision alternatives and states of nature.

Probability: Probability gives us a numerical measurement of the likelihood that an event will occur.

Regret criterion: A decision rule based on the fact that if a decision-maker selects an alternative that does not turn out to be the best, he or she will experience regret and therefore decisions should be made that will minimize the maximum possible regret.

Risk: A term applied to a situation where there are several possible outcomes and there is relevant past experience to enable statistical evidence to be produced for predicting the possible outcomes.

Risk Seeker : A risk seeker is a decision maker who is interested in the best outcomes no matter how small the chance that they may occur.

Risk averse decision maker: A risk averse decision maker acts on the assumption that the worst outcome might occur.

Risk management: It is a systematic process of identifying and assessing company risks and taking actions to protect a company against them.

Standard deviation: The square root of the mean of the squared deviations from the expected value.

States of nature: In the context of risk and uncertainty, factors that are outside the decision maker's control, also known as events.

Subjective probabilities: Probabilities that are based on an individual's expert knowledge, past experience, and on observations of current variables which are likely to affect future events.

Uncertainty: A term applied to a situation where there are several possible outcomes and but there is little previous statistical evidence to enable probabilities to be attached to possible outcomes.

NOTES